Buckley's *The Continuity Debate* is a highly original and insightful study of one of the most puzzling subjects in the world, the continuity of numbers in real analysis. There is a close study of the contributions of the great mathematicians and philosophers Richard Dedekind, Georg Cantor, Paul du Bois-Reymond, and Charles Sanders Peirce. Buckley arranges the treatment of these figures to make a forceful philosophical argument defending the role of infinitesimals, and du Bois-Reymond emerges as the hero of the theory of continuity. The book explores the overall significance of the treatment of continuity and of infinitesimals for mathematics and the philosophy of mathematics. Despite the conceptual and mathematical difficulty of the issues, the book is accessible to readers without technical knowledge, and it is beautifully written. This is essential reading for anyone interested in the perplexing question what the continuity of numbers, magnitudes, and the geometrical line might amount to.

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Buckley's book offers a welcome perspective for a historical understanding of a synthetic continuum where infinitesimals acquire a sound life, according to Robinson's nonstandard analysis. His extensive perusal of infinitesimals in Du Bois-Reymond and Peirce, in particular, show how much we have lost through purely analytical tools for measuring continuity. Buckley explores a rich melting between history, philosophy, foundations and mathematics, through which the latter emerges well enriched. A much needed monograph, in times when mathematical imagination is breathing with force again.

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Docent Press/www.docentpress.com
Cover Design by Brenda Riddell, Graphic Details, Portsmouth, NH

Docent Press

ISBN 9780983700487
90000 >
9 780983 700487

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