

"Jinqiao Duan's book introduces the reader to the actively developing theory of stochastic dynamics through well-chosen examples that provide an overview, useful insights, and intuitive understanding of an often technically complicated topic."

– P. E. Kloeden, Goethe University, Frankfurt am Main

"Randomness is an important component of modeling complex phenomena in biological, chemical, physical, and engineering systems. Based on many years teaching this material, Jinqiao Duan develops a modern approach to the fundamental theory and application of stochastic dynamical systems for applied mathematicians and quantitative engineers and scientists. The highlight is the staged development of invariant stochastic structures that underpin much of our understanding of nonlinear stochastic systems and associated properties such as escape times. The book ranges from classic Brownian motion to noise generated by α -stable Levy flights."

– A. J. Roberts, University of Adelaide

This book serves as a concise introductory text on stochastic dynamics for applied mathematicians and scientists. Starting from the knowledge base typical for beginning graduate students in applied mathematics, it introduces the basic tools from probability and analysis and then develops for stochastic systems the properties traditionally calculated for deterministic systems. The book's final chapter opens the door to modeling in non-Gaussian situations, typical of many real-world applications. Rich with examples, illustrations, and exercises with solutions, this book is also ideal for self-study.

CAMBRIDGE TEXTS IN APPLIED MATHEMATICS

EDITORIAL BOARD

Professor M. J. Ablowitz, *Department of Applied Mathematics, University of Colorado, Boulder, USA*

Professor S. Davis, *School of Engineering Sciences and Applied Mathematics, Northwestern University, USA*

Professor E. J. Hinch, *Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK*

Professor A. Iserles, *Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK*

Dr J. Ockendon, *Mathematical Institute, University of Oxford, UK*

Professor P. J. Olver, *School of Mathematics, University of Minnesota, USA*

The aim of this series is to provide a focus for publishing textbooks in applied mathematics at the advanced undergraduate and beginning graduate level. It is planned that the books will be devoted to covering certain mathematical techniques and theories and exploring their applications.

Image: Pacific Storm Pair at 1415 UTC on 16 July 2012. Westbound Hurricane Fabio is aligned with an eastbound storm over the Pacific Northwest, with both swirls highlighted by the dawn. NASA-Goddard Space Flight Center, data from NOAA GOES.

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-1-107-42820-1



9 781107 428201 >

Contents

<i>Preface</i>	<i>page xi</i>
<i>Notation</i>	<i>xv</i>
1 Introduction	1
1.1 Examples of Deterministic Dynamical Systems	1
1.2 Examples of Stochastic Dynamical Systems	9
1.3 Mathematical Modeling with Stochastic Differential Equations	11
1.4 Outline of This Book	11
1.5 Problems	13
2 Background in Analysis and Probability	14
2.1 Euclidean Space	14
2.2 Hilbert, Banach, and Metric Spaces	15
2.3 Taylor Expansions	16
2.4 Improper Integrals and Cauchy Principal Values	17
2.5 Some Useful Inequalities	19
2.6 Hölder Spaces, Sobolev Spaces, and Related Inequalities	22
2.7 Probability Spaces	27
2.8 Stochastic Processes	35
2.9 Convergence Concepts	38
2.10 Simulation	39
2.11 Problems	41
3 Noise	44
3.1 Brownian Motion	44
3.2 What Is Gaussian White Noise?	50
3.3 *A Mathematical Model for Gaussian White Noise	52

3.4	Simulation	56
3.5	Problems	58
4	A Crash Course in Stochastic Differential Equations	61
4.1	Differential Equations with Noise	61
4.2	Riemann-Stieltjes Integration	62
4.3	Stochastic Integration and Stochastic Differential Equations	63
4.4	Itô's Formula	75
4.5	Linear Stochastic Differential Equations	82
4.6	Nonlinear Stochastic Differential Equations	88
4.7	Conversion between Itô and Stratonovich Stochastic Differential Equations	89
4.8	Impact of Noise on Dynamics	92
4.9	Simulation	94
4.10	Problems	95
5	Deterministic Quantities for Stochastic Dynamics	99
5.1	Moments	100
5.2	Probability Density Functions	102
5.3	Most Probable Phase Portraits	111
5.4	Mean Exit Time	118
5.5	Escape Probability	123
5.6	Problems	129
6	Invariant Structures for Stochastic Dynamics	133
6.1	Deterministic Dynamical Systems	134
6.2	Measurable Dynamical Systems	148
6.3	Random Dynamical Systems	150
6.4	Linear Stochastic Dynamics	160
6.5	*Random Invariant Manifolds	170
6.6	Problems	180
7	Dynamical Systems Driven by Non-Gaussian Lévy Motions	188
7.1	Modeling via Stochastic Differential Equations with Lévy Motions	189
7.2	Lévy Motions	190
7.3	The α -Stable Lévy Motions	197
7.4	Stochastic Differential Equations with Lévy Motions	214
7.5	Mean Exit Time	221

7.6	Escape Probability and Transition Phenomena	228
7.7	Fokker-Planck Equations	237
7.8	Problems	241
<i>Hints and Solutions</i>		245
<i>Further Readings</i>		271
<i>References</i>		273
<i>Index</i>		287