

Actuarial Modelling of Claim Counts

Risk Classification, Credibility and Bonus-Malus Systems

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There are a wide range of variables for actuaries to consider when calculating a motorist's insurance premium, such as age, gender and type of vehicle. Further to these factors, motorists' rates are subject to experience rating systems, including credibility mechanisms and Bonus Malus systems (BMSs).

Actuarial Modelling of Claim Counts presents a comprehensive treatment of the various experience rating systems and their relationships with risk classification. The authors summarize the most recent developments in the field, presenting ratemaking systems, whilst taking into account exogenous information.

- Offers the first self-contained, practical approach to a priori and a posteriori ratemaking in motor insurance.
- Discusses the issues of claim frequency and claim severity, multi-event systems, and the combinations of deductibles and BMSs.
- Introduces recent developments in actuarial science and exploits the generalised linear model and generalised linear mixed model to achieve risk classification.
- Presents credibility mechanisms as refinements of commercial BMSs.
- Provides practical applications with real data sets processed with SAS software.

Actuarial Modelling of Claim Counts is essential reading for students in actuarial science, as well as practicing and academic actuaries. It is also ideally suited for professionals involved in the insurance industry, applied mathematicians, quantitative economists, financial engineers and statisticians.

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