

This book is an introduction to the field of asymptotic statistics. The treatment is both practical and mathematically rigorous. In addition to most of the standard topics of an asymptotics course, including likelihood inference,  $M$ -estimation, the theory of asymptotic efficiency,  $U$ -statistics, and rank procedures, the book presents recent research topics such as semiparametric models, the bootstrap, and empirical processes and their applications.

The topics are organized from the central idea of approximation by limit experiments, which gives the book one of its unifying themes. This entails mainly the local approximation of the classical i.i.d. setup with smooth parameters by location experiments involving a single, normally distributed observation. Thus, even the standard subjects of asymptotic statistics are presented in a novel way.

Suitable as a text for a graduate or Master's level statistics course, this book will also give researchers in statistics, probability, and their applications an overview of the latest research in asymptotic statistics.

"I am very impressed by the author's success of keeping every chapter relatively short while the main ideas are discussed clearly with rigorous mathematical treatment. Many examples are provided to help in understanding the theory as well as to illustrate how asymptotic results are adopted in statistical applications.... Since asymptotic study is a crucial part of statistical research, this book will become a valuable tool for researchers like myself. It is also definitely an excellent textbook for at least two one-semester courses in asymptotic statistics, and I will certainly use it next time I teach such a course."

— Jun Shao, *University of Wisconsin*

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# Contents

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<i>Preface</i>		<i>page</i> xiii
<i>Notation</i>		<i>page</i> xv
1. Introduction		1
1.1. Approximate Statistical Procedures		1
1.2. Asymptotic Optimality Theory		2
1.3. Limitations		3
1.4. The Index $n$		4
2. Stochastic Convergence		5
2.1. Basic Theory		5
2.2. Stochastic $o$ and $O$ Symbols		12
*2.3. Characteristic Functions		13
*2.4. Almost-Sure Representations		17
*2.5. Convergence of Moments		17
*2.6. Convergence-Determining Classes		18
*2.7. Law of the Iterated Logarithm		19
*2.8. Lindeberg-Feller Theorem		20
*2.9. Convergence in Total Variation		22
Problems		24
3. Delta Method		25
3.1. Basic Result		25
3.2. Variance-Stabilizing Transformations		30
*3.3. Higher-Order Expansions		31
*3.4. Uniform Delta Method		32
*3.5. Moments		33
Problems		34
4. Moment Estimators		35
4.1. Method of Moments		35
*4.2. Exponential Families		37
Problems		40
5. $M$ - and $Z$ -Estimators		41
5.1. Introduction		41
5.2. Consistency		44
5.3. Asymptotic Normality		51



*5.4.	Estimated Parameters	60
5.5.	Maximum Likelihood Estimators	61
*5.6.	Classical Conditions	67
*5.7.	One-Step Estimators	71
*5.8.	Rates of Convergence	75
*5.9.	Argmax Theorem	79
	Problems	83
6.	Contiguity	85
6.1.	Likelihood Ratios	85
6.2.	Contiguity	87
	Problems	91
7.	Local Asymptotic Normality	92
7.1.	Introduction	92
7.2.	Expanding the Likelihood	93
7.3.	Convergence to a Normal Experiment	97
7.4.	Maximum Likelihood	100
*7.5.	Limit Distributions under Alternatives	103
*7.6.	Local Asymptotic Normality	103
	Problems	106
8.	Efficiency of Estimators	108
8.1.	Asymptotic Concentration	108
8.2.	Relative Efficiency	110
8.3.	Lower Bound for Experiments	111
8.4.	Estimating Normal Means	112
8.5.	Convolution Theorem	115
8.6.	Almost-Everywhere Convolution	
	Theorem	115
*8.7.	Local Asymptotic Minimax Theorem	117
*8.8.	Shrinkage Estimators	119
*8.9.	Achieving the Bound	120
*8.10.	Large Deviations	122
	Problems	123
9.	Limits of Experiments	125
9.1.	Introduction	125
9.2.	Asymptotic Representation Theorem	126
9.3.	Asymptotic Normality	127
9.4.	Uniform Distribution	129
9.5.	Pareto Distribution	130
9.6.	Asymptotic Mixed Normality	131
9.7.	Heuristics	136
	Problems	137
10.	Bayes Procedures	138
10.1.	Introduction	138
10.2.	Bernstein-von Mises Theorem	140

10.3. Point Estimators	146
*10.4. Consistency	149
Problems	152
11. Projections	153
11.1. Projections	153
11.2. Conditional Expectation	155
11.3. Projection onto Sums	157
*11.4. Hoeffding Decomposition	157
Problems	160
12. $U$ -Statistics	161
12.1. One-Sample $U$ -Statistics	161
12.2. Two-Sample $U$ -statistics	165
*12.3. Degenerate $U$ -Statistics	167
Problems	171
13. Rank, Sign, and Permutation Statistics	173
13.1. Rank Statistics	173
13.2. Signed Rank Statistics	181
13.3. Rank Statistics for Independence	184
*13.4. Rank Statistics under Alternatives	184
13.5. Permutation Tests	188
*13.6. Rank Central Limit Theorem	190
Problems	190
14. Relative Efficiency of Tests	192
14.1. Asymptotic Power Functions	192
14.2. Consistency	199
14.3. Asymptotic Relative Efficiency	201
*14.4. Other Relative Efficiencies	202
*14.5. Rescaling Rates	211
Problems	213
15. Efficiency of Tests	215
15.1. Asymptotic Representation Theorem	215
15.2. Testing Normal Means	216
15.3. Local Asymptotic Normality	218
15.4. One-Sample Location	220
15.5. Two-Sample Problems	223
Problems	226
16. Likelihood Ratio Tests	227
16.1. Introduction	227
*16.2. Taylor Expansion	229
16.3. Using Local Asymptotic Normality	231
16.4. Asymptotic Power Functions	236



16.5. Bartlett Correction	238
*16.6. Bahadur Efficiency	238
Problems	241
17. Chi-Square Tests	242
17.1. Quadratic Forms in Normal Vectors	242
17.2. Pearson Statistic	242
17.3. Estimated Parameters	244
17.4. Testing Independence	247
*17.5. Goodness-of-Fit Tests	248
*17.6. Asymptotic Efficiency	251
Problems	253
18. Stochastic Convergence in Metric Spaces	255
18.1. Metric and Normed Spaces	255
18.2. Basic Properties	258
18.3. Bounded Stochastic Processes	260
Problems	263
19. Empirical Processes	265
19.1. Empirical Distribution Functions	265
19.2. Empirical Distributions	269
19.3. Goodness-of-Fit Statistics	277
19.4. Random Functions	279
19.5. Changing Classes	282
19.6. Maximal Inequalities	284
Problems	289
20. Functional Delta Method	291
20.1. von Mises Calculus	291
20.2. Hadamard-Differentiable Functions	296
20.3. Some Examples	298
Problems	303
21. Quantiles and Order Statistics	304
21.1. Weak Consistency	304
21.2. Asymptotic Normality	305
21.3. Median Absolute Deviation	310
21.4. Extreme Values	312
Problems	315
22. $L$ -Statistics	316
22.1. Introduction	316
22.2. Hájek Projection	318
22.3. Delta Method	320
22.4. $L$ -Estimators for Location	323
Problems	324
23. Bootstrap	326

23.1. Introduction	326
23.2. Consistency	329
23.3. Higher-Order Correctness Problems	334
	339
24. Nonparametric Density Estimation	341
24.1 Introduction	341
24.2 Kernel Estimators	341
24.3 Rate Optimality	346
24.4 Estimating a Unimodal Density Problems	349
	356
25. Semiparametric Models	358
25.1 Introduction	358
25.2 Banach and Hilbert Spaces	360
25.3 Tangent Spaces and Information	362
25.4 Efficient Score Functions	368
25.5 Score and Information Operators	371
25.6 Testing	384
*25.7 Efficiency and the Delta Method	386
25.8 Efficient Score Equations	391
25.9 General Estimating Equations	400
25.10 Maximum Likelihood Estimators	402
25.11 Approximately Least-Favorable Submodels	408
25.12 Likelihood Equations Problems	419
	431
<i>References</i>	433
<i>Index</i>	439