This book is an introduction to the field of asymptotic statistics. The treatment is both practical and mathematically rigorous. In addition to most of the standard topics of an asymptotics course, including likelihood inference, *M*-estimation, the theory of asymptotic efficiency, *U*-statistics, and rank procedures, the book presents recent research topics such as semiparametric models, the bootstrap, and empirical processes and their applications.

The topics are organized from the central idea of approximation by limit experiments, which gives the book one of its unifying themes. This entails mainly the local approximation of the classical i.i.d. setup with smooth parameters by location experiments involving a single, normally distributed observation. Thus, even the standard subjects of asymptotic statistics are presented in a novel way.

Suitable as a text for a graduate or Master's level statistics course, this book will also give researchers in statistics, probability, and their applications an overview of the latest research in asymptotic statistics.

"I am very impressed by the author's success of keeping every chapter relatively short while the main ideas are discussed clearly with rigorous mathematical treatment. Many examples are provided to help in understanding the theory as well as to illustrate how asymptotic results are adopted in statistical applications.... Since asymptotic study is a crucial part of statistical research, this book will become a valuable tool for researchers like myself. It is also definitely an excellent textbook for at least two one-semester courses in asymptotic statistics, and I will certainly use it next time I teach such a course."

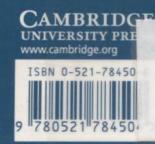
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