

CHAPTER V	Harmonic Functions	241
§8. Application: Flow Lines	241	
§9. Examples	252	
§10. Basic Properties of Harmonic Functions	259	
§11. The Poisson Formula	271	
§12. The Poisson Integral as a Convolution	273	
§13. Construction of Harmonic Functions	276	
§14. Appendix: Differentiating Under the Integral Sign	286	
§15. The Harnack Inequality for Convex Functions	289	
§16. Exercises to Chapter V	291	
PART TWO	Geometric Function Theory	291

CHAPTER VI	Complex Numbers and Complex Functions	v
Foreword		
Prerequisites		
PART ONE	Basic Theory	1
CHAPTER I	Complex Numbers and Functions	3
§1. Definition		3
§2. Polar Form		8
§3. Complex Valued Functions		12
§4. Limits and Compact Sets		17
Compact Sets		21
§5. Complex Differentiability		27
§6. The Cauchy–Riemann Equations		31
§7. Angles Under Holomorphic Maps		33
CHAPTER II	Power Series	37
§1. Formal Power Series		37
§2. Convergent Power Series		47
§3. Relations Between Formal and Convergent Series		60
Sums and Products		60
Quotients		64
Composition of Series		66
§4. Analytic Functions		68
§5. Differentiation of Power Series		72
CHAPTER III	Entire Functions	75
§1. The Use of Three Circles and the Maximum Modulus Principle		75
§2. Hermite Interpolation Formula		83
§3. Entire Functions with Rational Values		89
§4. The Phragmen–Lindelöf and Neumann Theorems		93

§6. The Inverse and Open Mapping Theorems	76
§7. The Local Maximum Modulus Principle	83
 CHAPTER III Cauchy's Theorem, First Part	
§1. Holomorphic Functions on Connected Sets	86
Appendix: Connectedness	92
§2. Integrals Over Paths	94
§3. Local Primitive for a Holomorphic Function	104
§4. Another Description of the Integral Along a Path	110
§5. The Homotopy Form of Cauchy's Theorem	115
§6. Existence of Global Primitives. Definition of the Logarithm	119
§7. The Local Cauchy Formula	125
 CHAPTER IV Winding Numbers and Cauchy's Theorem	
§1. The Winding Number	134
§2. The Global Cauchy Theorem	138
Dixon's Proof of Theorem 2.5 (Cauchy's Formula)	147
§3. Artin's Proof	149
 CHAPTER V Applications of Cauchy's Integral Formula	
§1. Uniform Limits of Analytic Functions	156
§2. Laurent Series	161
§3. Isolated Singularities	165
Removable Singularities	165
Poles	166
Essential Singularities	168
 CHAPTER VI Calculus of Residues	
§1. The Residue Formula	173
Residues of Differentials	184
§2. Evaluation of Definite Integrals	191
Fourier Transforms	194
Trigonometric Integrals	197
Mellin Transforms	199
 CHAPTER VII Conformal Mappings	
§1. Schwarz Lemma	208
§2. Analytic Automorphisms of the Disc	210
§3. The Upper Half Plane	212
§4. Other Examples	215
§5. Fractional Linear Transformations	220
	231

CHAPTER VIII		
Harmonic Functions		241
§1. Definition		241
Application: Perpendicularity		246
Application: Flow Lines		248
§2. Examples		252
§3. Basic Properties of Harmonic Functions		259
§4. The Poisson Formula		271
The Poisson Integral as a Convolution		273
§5. Construction of Harmonic Functions		276
§6. Appendix. Differentiating Under the Integral Sign		286
PART TWO		
Geometric Function Theory		291
CHAPTER IX		
Schwarz Reflection		293
§1. Schwarz Reflection (by Complex Conjugation)		293
§2. Reflection Across Analytic Arcs		297
§3. Application of Schwarz Reflection		303
CHAPTER X		
The Riemann Mapping Theorem		306
§1. Statement of the Theorem		306
§2. Compact Sets in Function Spaces		308
§3. Proof of the Riemann Mapping Theorem		311
§4. Behavior at the Boundary		314
CHAPTER XI		
Analytic Continuation Along Curves		322
§1. Continuation Along a Curve		322
§2. The Dilogarithm		331
§3. Application to Picard's Theorem		335
PART THREE		
Various Analytic Topics		337
CHAPTER XII		
Applications of the Maximum Modulus Principle and Jensen's Formula		339
§1. Jensen's Formula		340
§2. The Picard-Borel Theorem		346
§3. Bounds by the Real Part, Borel-Carathéodory Theorem		354
§4. The Use of Three Circles and the Effect of Small Derivatives		356
Hermite Interpolation Formula		358
§5. Entire Functions with Rational Values		360
§6. The Phragmen-Lindelöf and Hadamard Theorems		365

CHAPTER XIII	<i>and Open Mapping Theorems</i>	HY STRANG
Entire and Meromorphic Functions	Principle	372
§1. Infinite Products	372
§2. Weierstrass Products	376
§3. Functions of Finite Order	382
§4. Meromorphic Functions, Mittag-Leffler Theorem	387
CHAPTER XIV	<i>Connectedness</i>	HY STRANG
Elliptic Functions	391
§1. The Liouville Theorems	391
§2. The Weierstrass Function	395
§3. The Addition Theorem	400
§4. The Sigma and Zeta Functions	403
CHAPTER XV	<i>Path</i>	XI REST
The Gamma and Zeta Functions	408
§1. The Differentiation Lemma	409
§2. The Gamma Function	413
Weierstrass Product	413
The Gauss Multiplication Formula (Distribution Relation)	416
The (Other) Gauss Formula	418
The Mellin Transform	420
The Stirling Formula	422
Proof of Stirling's Formula	424
§3. The Lerch Formula	431
§4. Zeta Functions	433
CHAPTER XVI	<i>Integration</i>	XI REST
The Prime Number Theorem	440
§1. Basic Analytic Properties of the Zeta Function	441
§2. The Main Lemma and its Application	446
§3. Proof of the Main Lemma	449
Appendix	453
§1. Summation by Parts and Non-Absolute Convergence	453
§2. Difference Equations	457
§3. Analytic Differential Equations	461
§4. Fixed Points of a Fractional Linear Transformation	465
§5. Cauchy's Formula for C^∞ Functions	467
§6. Cauchy's Theorem for Locally Integrable Vector Fields	472
§7. More on Cauchy-Riemann	477
Bibliography	479
Index	481