From the Preface to the First Printing v
From the Preface to the Seventh Printing viii
Preface to the Second Edition ix
"How to Solve It" list xvi
Foreword xix
Introduction xxv

PART I. IN THE CLASSROOM

Purpose

1. Helping the student 1
2. Questions, recommendations, mental operations 1
3. Generality 2
4. Common sense 3
5. Teacher and student. Imitation and practice 3

Main divisions, main questions

6. Four phases 5
7. Understanding the problem 6
8. Example 7
9. Devising a plan 8
10. Example 10
11. Carrying out the plan 12

12. Example 13
13. Looking back 14
14. Example 16
15. Various approaches 19
16. The teacher's method of questioning 20
17. Good questions and bad questions 22

More examples

18. A problem of construction 23
19. A problem to prove 25
20. A rate problem 29

PART II. HOW TO SOLVE IT

A dialogue 33

PART III. SHORT DICTIONARY OF HEURISTIC

Analogy 37
Auxiliary elements 46
Auxiliary problem 50
Bolzano 57
Bright idea 58
Can you check the result? 59
Can you derive the result differently? 61
Can you use the result? 64
Carrying out 68

Condition 72
Contradictory† 73
Corollary 73
Could you derive something useful from the data? 73
Could you restate the problem?† 75
Decomposing and recombining 75
Definition 85
Descartes 92
Determination, hope, success 93
Diagnosis 94
Did you use all the data? 95
Do you know a related problem? 98
Draw a figure† 99
Examine your guess 99
Figures 103
Generalization 108
Have you seen it before? 110
Here is a problem related to yours and solved before 110
Heuristic 112
Heuristic reasoning 113
If you cannot solve the proposed problem 114
Induction and mathematical induction 114
Inventor's paradox 121
Is it possible to satisfy the condition? 122
Leibnitz 123
Lemma 123

† Contains only cross-references.

Look at the unknown 123
Modern heuristic 129
Notation 134
Pappus 141
Pedantry and mastery 148
Practical problems 149
Problems to find, problems to prove 154
Progress and achievement 157
Puzzles 160
Reductio ad absurdum and indirect proof 162
Redundant† 171
Routine problem 171
Rules of discovery 172
Rules of style 172
Rules of teaching 173
Separate the various parts of the condition 173
Setting up equations 174
Signs of progress 178
Specialization 190
Subconscious work 197
Symmetry 199
Terms, old and new 200
Test by dimension 202
The future mathematician 205
The intelligent problem-solver 206
The intelligent reader 207
The traditional mathematics professor 208

Variation of the problem 209
What is the unknown? 214
Why proofs? 215
Wisdom of proverbs 221
Working backwards 225

PART IV. PROBLEMS, HINTS, SOLUTIONS

Problems 234
Hints 238
Solutions 242