

Macroscopic Electrodynamics

An Introductory Graduate Treatment

"Macroscopic Electrodynamics" is a comprehensive two-semester introductory graduate-level textbook on classical electrodynamics for use in physics and engineering programs. The word "macroscopic" is intended to indicate both the large-scale nature of the theory, as well as the fact that emphasis is placed upon applications of the so-called macroscopic Maxwell equations to idealized media.

This book emphasizes principles and practical methods of analysis, which are often presented in fresh and original ways. Illustrative examples are carefully chosen to promote the students' physical intuition, and are worked out in detail to give students a thorough grounding in solution techniques. The style is informal yet mathematically sound, and presumes only a basic familiarity with electrodynamics such as may be obtained in a one-semester junior-level undergraduate class.

At the end of each chapter many original problems are provided which illustrate or expand upon specific sections of the text. The problems are at the heart of the text and are meant to encourage students, develop confidence, and emphasize ideas while avoiding both oversimplification and inordinate calculational difficulties.

World Scientific

www.worldscientific.com

9235 sc

ISBN 978-981-4616-62-1 (pbk)



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