

Contents

Preface	ix
1 Imaginative Words	1
1.1 Lemniscate	1
1.2 Centillion	3
1.3 Golden Ratio	3
1.4 Borromean Rings	5
1.5 Sieve of Eratosthenes	5
1.6 Transversal of Primes	6
1.7 Waterfall of Primes	7
1.8 Squares, Triangular Numbers, and Cubes	7
1.9 Determinant	8
1.10 Complex Plane	8
2 Intriguing Images	13
2.1 Square Pyramidal Square Number	13
2.2 Binary Trees	15
2.3 Bulging Hyperspheres	16
2.4 Projective Plane	16
2.5 Two-Colored Graph	17
2.6 Hypercube	18
2.7 Full Adder	19
2.8 Sierpiński's Triangle	20
2.9 Squaring Map	21
2.10 Riemann Sphere	22
3 Captivating Formulas	25
3.1 Arithmetical Wonders	25
3.2 Heron's Formula and Heronian Triangles	25
3.3 Sine, Cosine, and Exponential Function Expansions	28
3.4 Tangent and Secant Function Expansions	29
3.5 Series for Pi	30
3.6 Product for Pi	31
3.7 Fibonacci Numbers and Pi	32
3.8 Volume of a Ball	32
3.9 Euler's Integral Formula	34
3.10 Euler's Polyhedral Formula	35

3.11 The Smallest Taxicab Number	36
3.12 Infinity and Infinity Squared	37
3.13 Complex Functions	38
3.14 The Zeta Function and Bernoulli Numbers	40
3.15 The Riemann Zeta Function	41
3.16 The Jacobi Identity	42
3.17 Entropy	43
3.18 Rook Paths	44
4 Delightful Theorems	49
4.1 A Square inside Every Triangle	49
4.2 Morley's Theorem	50
4.3 The Euler Line	52
4.4 Monge's Theorem	54
4.5 Power Means	54
4.6 Regular Heptagon	58
4.7 Isometries of the Plane	59
4.8 Symmetries of Regular Convex Polyhedra	61
4.9 Polynomial Symmetries	63
4.10 Kings and Serfs	65
4.11 The Erdős-Szekeres Theorem	66
4.12 Minkowski's Theorem	67
4.13 Lagrange's Theorem	69
4.14 Van der Waerden's Theorem	72
4.15 Latin Squares and Projective Planes	76
4.16 The Lemniscate Revisited	79
5 Pleasing Proofs	83
5.1 The Pythagorean Theorem	83
5.2 The Erdős-Mordell Inequality	84
5.3 Triangles with Given Area and Perimeter	85
5.4 A Property of the Directrix of a Parabola	86
5.5 A Classic Integral	87
5.6 Integer Partitions	88
5.7 Integer Triangles	89
5.8 Triangle Destruction	92
5.9 Squares in Arithmetic Progression	94
5.10 Random Hemispheres	95
5.11 Odd Binomial Coefficients	95
5.12 Frobenius' Postage Stamp Problem	96
5.13 Perrin's Sequence	99
5.14 On the Number of Partial Orders	99
5.15 Perfect Error-Correcting Codes	101
5.16 Binomial Coefficient Magic	104
5.17 A Group of Operations	106

6	Elegant Solutions	109
6.1	A Tetrahedron and Four Spheres	109
6.2	Alphabet Cubes	110
6.3	A Triangle in an Ellipse	110
6.4	About the Roots of a Cubic	111
6.5	Distance on Planet X	113
6.6	A Tilted Circle	114
6.7	The Millionth Fibonacci Number	116
6.8	The End of a Conjecture	117
6.9	A Zero-Sum Game	117
6.10	An Expected Maximum	119
6.11	Walks on a Graph	120
6.12	Rotations of a Grid	123
6.13	Stamp Rolls	125
6.14	Making a Million	128
6.15	Coloring a Projective Plane	129
7	Creative Problems	131
7.1	Two-Dimensional Gobbling Algorithm	131
7.2	Nonattacking Queens Game	132
7.3	Lucas Numbers Mod m	132
7.4	Exact Colorings of Graphs	133
7.5	Queen Paths	134
7.6	Transversal Achievement Game	136
7.7	Binary Matrix Game	136
A	Harmonious Foundations	139
A.1	Sets	139
A.2	Relations	141
A.3	Functions	141
A.4	Groups	142
A.5	Fields	145
A.6	Vector Spaces	146
B	Eye-Opening Explorations	151
B.1	Problems	151
B.2	Solutions	155
	Bibliography	165
	Index	169
	About the Author	177