

This book provides an extensive survey of all the physics necessary for understanding current developments in the field of fundamental cosmology, as well as the observational data and methods. It will help students to get into research, with definitions and main techniques and ideas discussed today. The book is divided into three parts. Part 1 summarises the fundamentals in theoretical physics needed in cosmology (general relativity, field theory, particle physics). Part 2 describes the standard model of cosmology and includes cosmological solutions of Einstein equations, the hot big bang model, cosmological perturbation theory, cosmic microwave background anisotropies, lensing and evidence for dark matter, and inflation. Part 3 describes extensions of this model and opens up current research in the field: scalar-tensor theories, supersymmetry, the cosmological constant problem and acceleration of the universe, topology of the universe, grand unification and baryogenesis, topological defects and phase transitions, string inspired cosmology including branes and the latest developments. The book provides details of all derivations and leads the student up to the level of research articles.

Patrick Peter and **Jean-Philippe Uzan** are at the Institut d'Astrophysique de Paris, France.

'Fills a niche that other recent cosmology texts leave open, namely self-contained derivations in cosmology that span both fundamental issues and applications to the real universe that are of great interest to observers.'

Joseph Silk, University of Oxford

'A remarkable book. Written with great authority and enthusiasm, it gives a comprehensive view of primordial cosmology today. The style is accessible to a novice for a good introduction, as well as being technically precise enough to be useful for specialists.'

Ted Jacobson, University of Maryland

Cover image: 'Le vent' by Madeleine Attal (1997).

**ALSO AVAILABLE FROM
OXFORD UNIVERSITY PRESS**

Cosmology
Steven Weinberg

Particle Astrophysics
Second Edition
D.H. Perkins

Conceptions of Cosmos
From Myths to the Accelerating Universe: A History of Cosmology
Helge S. Kragh

Introduction to Black Hole Physics
Valeri P. Frolov, Andrei Zelnikov

OXFORD
UNIVERSITY PRESS

www.oup.com

ISBN 978-0-19-966515-0



9 780199 665150

Contents

Foreword to the English edition	v
Foreword to the French edition	vii
Acknowledgements	ix
Introduction	1
Cosmology: an ancient yet contemporary subject	1
The specifics of cosmology	2
Primordial cosmology	2
Organization and objective of this book	3
Warnings	5
References	6
PART I OVERVIEW OF THE THEORETICAL BASIS	
1 General relativity	11
1.1 Space-time and gravity	11
1.1.1 Absolute space and time of Newtonian physics	11
1.1.2 Space-time of special relativity	14
1.1.3 General relativity and curved space-time	19
1.2 Elements of differential geometry	21
1.2.1 Manifolds and tensors	21
1.2.2 Geodesic equations and Christoffel symbols	26
1.2.3 Covariant derivative, parallel transport and Lie derivative	28
1.2.4 Curvature	31
1.2.5 Covariant approach	34
1.3 Equations of motion	38
1.3.1 Einstein's equations	38
1.3.2 Conservation equations	41
1.3.3 ADM Hamiltonian formulation	44
1.4 Geodesics in a curved space-time	46
1.4.1 Conserved quantities along a geodesic	46
1.4.2 Geodesic deviation equation	47
1.5 Weak-field regime	49
1.5.1 Newtonian limit	49
1.5.2 Gravitational waves in an empty space-time	53
1.6 Tests of general relativity	55
1.6.1 Test of the equivalence principle	55
1.6.2 Tests in the Solar System	60

1.6.3	Gravitational radiation	62
1.6.4	Need for tests at astrophysical scales	65
	References	66
2	Overview of particle physics and the Standard Model	68
2.1	From classical to quantum	68
2.1.1	Analytical classical mechanics	69
2.1.2	Quantum physics	79
2.1.3	Quantum and relativistic mechanics	86
2.2	Canonical decompositions	89
2.2.1	Technical clarification	89
2.2.2	Real free field	90
2.2.3	The complex scalar field	95
2.3	Classification and properties of the elementary particles	98
2.4	Internal symmetries	101
2.4.1	Group theory	101
2.4.2	Generators	103
2.5	Symmetry breaking	106
2.5.1	Gauge group	107
2.5.2	Higgs mechanism	108
2.5.3	Non-Abelian case	110
2.6	The standard model $SU(3)_c \times SU(2)_L \times U(1)_Y$	112
2.6.1	The strong interaction (QCD)	113
2.6.2	Electroweak interaction	114
2.6.3	A complete model	116
2.7	Discrete invariances	121
2.7.1	Parity	121
2.7.2	Charge conjugation	123
2.7.3	Time reversal and CPT theorem	124
	References	126
	PART II THE MODERN STANDARD COSMOLOGICAL MODEL	
3	The homogeneous Universe	129
3.1	The cosmological solution of Friedmann–Lemaître	129
3.1.1	Constructing a Universe model	129
3.1.2	Cosmological and Copernican principles	130
3.1.3	Cosmological principle and metric	131
3.1.4	Kinematics	136
3.1.5	Space-time dynamics	138
3.2	Dynamics of Friedmann–Lemaître space-times	143
3.2.1	Some solutions	143
3.2.2	Dynamical evolution	144
3.2.3	Expansion and contraction	148
3.3	Time and distances	150
3.3.1	Age of the Universe and look-back time	150

3.3.2	Comoving radial distance	151
3.3.3	Angular distances	152
3.3.4	Luminosity distance	154
3.3.5	Volume and number counts	157
3.4	Behaviour at small redshifts	158
3.4.1	Deceleration parameter	158
3.4.2	Expression of the time and distances	158
3.5	Horizons	160
3.5.1	Event horizon	160
3.5.2	Particle horizon	162
3.5.3	Global properties	163
3.6	Beyond the cosmological principle	167
3.6.1	Classifying space-times	168
3.6.2	Universe with non-homogeneous spatial sections	170
3.6.3	Universe with homogeneous spatial sections	171
	References	175
4	The standard Big-Bang model	177
4.1	The Hubble diagram and the age of the Universe	177
4.1.1	The Hubble constant	178
4.1.2	The contribution of supernovæ	182
4.1.3	The age of the Universe	186
4.2	Thermodynamics in an expanding Universe	188
4.2.1	Equilibrium thermodynamics	188
4.2.2	Out-of-equilibrium thermodynamics	197
4.2.3	Two limiting cases	201
4.3	Primordial nucleosynthesis	204
4.3.1	Main stages of the mechanism	205
4.3.2	Initial state	206
4.3.3	Freeze-out of the weak interaction and neutron to proton ratio	208
4.3.4	Abundances of the light elements	211
4.3.5	Observational status	215
4.4	The cosmic microwave background radiation	216
4.4.1	Recombination	217
4.4.2	Properties of the cosmic microwave background	221
4.4.3	Another proof of the expansion of the Universe	226
4.5	Status of the Big-Bang model	226
4.5.1	A good standard model...	226
4.5.2	... but an incomplete model	228
4.5.3	Conclusion	234
	References	235

5	The inhomogeneous Universe	238
5.1	Newtonian perturbations	238
5.1.1	Case of a static space	238
5.1.2	Case of an expanding space	240
5.1.3	Predictions and observables	246
5.1.4	Towards the non-linear regime	249
5.2	Gauge invariant cosmological perturbation theory	251
5.2.1	Perturbed space-time	251
5.2.2	Description of matter	255
5.2.3	Choosing a gauge	257
5.2.4	Einstein equations: derivation	260
5.2.5	Einstein equations: SVT decomposition	262
5.2.6	Perturbed conservation equation for a fluid	264
5.2.7	Interpretation of the perturbation equations	265
5.3	Evolution	266
5.3.1	Vector and tensor modes	267
5.3.2	Evolution of the gravitational potential	269
5.3.3	Scalar modes in the adiabatic regime	272
5.3.4	Mixture of several fluids	278
5.4	Power spectrum of density fluctuations	282
5.4.1	Two equivalent approaches	283
5.4.2	Different regimes	285
5.4.3	Some refinements	292
5.5	The large-scale structure of the Universe	296
5.5.1	Observing the large-scale structure	296
5.5.2	Structures in the non-linear regime	299
	References	304
6	The cosmic microwave background	307
6.1	Origin of the cosmic microwave background anisotropies	307
6.1.1	Sachs–Wolfe formula	307
6.1.2	Angular power spectrum	311
6.2	Properties of the angular power spectrum	316
6.2.1	Large angular scales	317
6.2.2	Intermediate scales	318
6.2.3	Small scales	323
6.2.4	Other effects	326
6.3	Kinetic description	329
6.3.1	Perturbed Boltzmann equation	330
6.3.2	Gauge invariant expressions	339
6.3.3	Thomson scattering and polarization	342
6.3.4	Numerical integration	355
6.4	Anisotropies of the cosmic microwave background	356
6.5	Effects of the parameters on the angular power spectrum	364
6.5.1	Cosmological parameters	364
6.5.2	Parameters describing the primordial physics	367

References	371
7 Gravitational lensing and dark matter	374
7.1 Gravitational lensing and its applications	374
7.1.1 Gravitational lensing in the thin-lens regime	375
7.1.2 Lensing by galaxies and galaxy clusters	387
7.1.3 Gravitational distortion by the large-scale structure	395
7.1.4 Cosmic convergence	399
7.1.5 Cosmic shear	402
7.1.6 Measurement of the cosmic shear	405
7.1.7 Lensing on the cosmic microwave background	408
7.2 Evidence for the existence of dark matter	411
7.2.1 Dark matter in galaxies	411
7.2.2 Dark matter in clusters and groups of galaxies	419
7.2.3 Cosmological evidence	422
7.2.4 Summary	423
7.2.5 Candidates and constraints	423
References	445
8 Inflation	450
8.1 Genesis of the paradigm	451
8.1.1 Original motivations	451
8.1.2 Resolution of the Big-Bang problems	452
8.1.3 First models of inflation	454
8.1.4 Inflation as a de Sitter phase	455
8.2 Dynamics of single-field inflation	457
8.2.1 Equations of evolution	457
8.2.2 Slow-roll parameters	459
8.2.3 End of inflation	463
8.2.4 Some examples	465
8.2.5 Classification of inflationary models	469
8.3 Quantum fluctuations during inflation	470
8.3.1 Massless test scalar field in a de Sitter space-time	470
8.3.2 Massive test field in de Sitter	476
8.3.3 Massive test field during slow-roll inflation	477
8.4 Quantum fluctuations of the inflaton	478
8.4.1 Overview of the questions to address and expected results	479
8.4.2 Perturbed quantities	479
8.4.3 Perturbation equations	481
8.4.4 Evolution of the long-wavelength modes	483
8.4.5 Junction conditions and their applications	485
8.4.6 Quantization of the density perturbations	488
8.4.7 Gravitational waves	492
8.5 Perturbations in the slow-roll regime	494
8.5.1 An exact solution: power-law inflation	495
8.5.2 General case	496

8.5.3	Relation to observations	501
8.5.4	Reconstruction of the potential	505
8.6	End of inflation and reheating	507
8.6.1	Perturbative reheating	507
8.6.2	Theory of preheating	510
8.7	Eternal inflation	516
8.7.1	Heuristic argument	516
8.7.2	Stochastic approach	518
8.8	Extensions	523
8.8.1	Multifield inflation	523
8.8.2	Non-Gaussianity	530
8.8.3	Trans-Planckian problem	532
8.9	Status of the paradigm	537
	References	541
PART III BEYOND THE STANDARD MODELS		
9	Grand unification and baryogenesis	549
9.1	Interactions	549
9.1.1	Superposition principle	550
9.1.2	N -particle states	551
9.2	Grand unification	554
9.2.1	Problems of the standard model	554
9.2.2	Unification of the coupling constants	560
9.2.3	Unification models	562
9.2.4	Consequences of grand unification	566
9.2.5	Neutrino mass	568
9.3	Baryogenesis	569
9.3.1	Sakharov conditions for baryogenesis	569
9.3.2	Electroweak anomalies	571
9.3.3	Electroweak baryogenesis	573
9.3.4	Leptogenesis	575
9.3.5	Affleck–Dine mechanism	576
	References	578
10	Extensions of the theoretical framework	579
10.1	Scalar-tensor theory of gravity	579
10.1.1	Formulation	580
10.1.2	Local constraints	584
10.1.3	Cosmological aspects	584
10.1.4	Phenomenological aspects	588
10.1.5	$f(R)$ gravity and scalar-tensor theory	593
10.2	Quantum field theory in curved space-time	594
10.2.1	Quantum physics, classical gravity	594
10.2.2	Particle creation	597
10.2.3	A complete example	602

10.3	Supersymmetry and supergravity	604
10.3.1	Technical generalities	604
10.3.2	Wess–Zumino model	610
10.3.3	Gauge field	614
10.3.4	Supersymmetry breaking	615
10.3.5	The minimal supersymmetric standard model (MSSM)	618
10.3.6	Supergravity	621
	References	625
11	Phase transitions and topological defects	628
11.1	Phase transitions	628
11.1.1	Thermal field theory	628
11.1.2	Dynamics of the symmetry breaking	630
11.1.3	Formation of topological defects	633
11.2	Domain walls	635
11.2.1	Correlation length	635
11.2.2	Static configurations	638
11.3	Vortices	640
11.3.1	Kibble mechanism for cosmic strings	641
11.3.2	Internal structure	643
11.4	Monopoles	647
11.5	Textures	648
11.6	Defects in general	648
11.6.1	Connectedness	649
11.6.2	Fundamental group	649
11.6.3	Homotopy group	650
11.6.4	Semi-topological defects	651
11.7	Walls in cosmology	653
11.7.1	Distribution and evolution	653
11.7.2	Observational constraints	655
11.8	Cosmological monopoles	656
11.8.1	GUT monopoles are unavoidable	656
11.8.2	The monopole problem	656
11.8.3	Possible solutions to the monopole problem	660
11.9	Cosmic strings	661
11.9.1	General properties	661
11.9.2	Gravitational effects of strings	662
11.9.3	Effect of a network on the microwave background	666
11.9.4	Other consequences of cosmic strings	671
	References	674
12	Cosmological extensions	678
12.1	Construction of inflationary models	678
12.1.1	A consistent model: supergravity	678
12.1.2	F -term inflation	681
12.1.3	D -term inflation	683

12.2	Cosmological constant and dark energy	684
12.2.1	The cosmological constant problem	685
12.2.2	The nature of dark energy	687
12.2.3	Quintessence	690
12.2.4	Other models	697
12.2.5	Other approaches	701
12.2.6	Parameterization of the equation of state	705
12.2.7	Implications for the formation of the large-scale structure	708
12.3	Varying constants	711
12.4	Topology of the Universe	716
12.4.1	Local and global structures	716
12.4.2	Mathematical introduction	718
12.4.3	Classification of the three-dimensional manifolds	718
12.4.4	Observational signatures	726
	References	733
13	Advanced topics	737
13.1	Extra dimensions: Kaluza–Klein theory	737
13.1.1	General relativity in D dimensions	737
13.1.2	Projected equations in four dimensions	739
13.1.3	Dilaton and Einstein–Maxwell theory	741
13.1.4	Einstein frame	742
13.1.5	Compactification	743
13.2	A few words on string theory	744
13.2.1	From particles to strings	744
13.2.2	Superstrings	746
13.2.3	Open and closed strings	749
13.2.4	Dualities	752
13.2.5	Low-energy Lagrangians	753
13.2.6	Origin of three space-like dimensions	755
13.3	The Universe as a ‘brane’	758
13.3.1	Motivations	758
13.3.2	Induced Einstein equations	761
13.3.3	The Randall–Sundrum model	764
13.3.4	Cosmological phenomenology	767
13.3.5	Possible extensions	768
13.3.6	The Universe as a defect	769
13.3.7	Models of induced gravity	771
13.4	Initial singularity and a bouncing Universe	772
13.4.1	Approaching the singularity	772
13.4.2	The pre-Big-Bang scenario	775
13.4.3	The cyclic scenarios	781
13.4.4	Regular bounce and power spectrum	785
	References	793

Appendix A Numerical values	796
A.1 Physical constants	796
A.2 Astrophysical quantities	796
A.3 Units	797
A.3.1 Natural units	797
A.3.2 Conversion factors	798
A.4 Particle physics	798
A.5 Cosmological quantities	799
A.6 Electromagnetic spectrum	799
References	803
Appendix B Special functions	804
B.1 Euler functions	804
B.2 Spherical harmonics	805
B.2.1 Definition	805
B.2.2 Expressions	805
B.2.3 Properties	805
B.2.4 Fourier transform	806
B.2.5 Useful integrals	807
B.3 Bessel functions	808
B.3.1 Definition	808
B.3.2 Asymptotic properties	808
B.3.3 Special cases	809
B.3.4 Spherical Bessel functions	809
B.3.5 Some useful integrals	810
B.4 Legendre polynomials	811
B.4.1 Associated Legendre polynomials	811
B.4.2 Legendre polynomials	811
B.5 Fourier transform and the eigenmodes of the Laplacian	812
B.5.1 Fourier transform	812
B.5.2 Power spectrum	812
B.5.3 Cartesian coordinates	813
B.5.4 Spherical coordinates	813
References	815
Appendix C Useful cosmological quantities	816
C.1 Background space	816
C.1.1 Geometry	816
C.1.2 Matter	817
C.2 Perturbed quantities	819
C.2.1 Geometry	819
C.2.2 Matter	821
Index	823