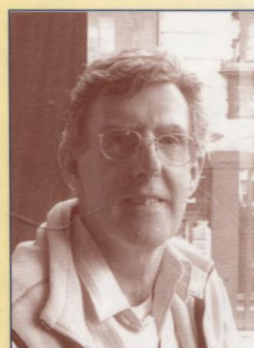


For two millennia the *right* way to teach geometry was the Euclidean approach, and in many respects, this is still the case. But in the 1950s, the cry "Down with triangles!" was heard in France and new geometry books appeared, packed with linear algebra but with no diagrams. Was this the new *right* approach? Or was the *right* approach still something else, perhaps transformation groups?



The Four Pillars of Geometry approaches geometry in four different ways, devoting two chapters to each. This makes the subject accessible to readers of all mathematical tastes, from the visual to the algebraic. Not only does each approach offer a different view; the combination of viewpoints yields insights not available in most books at this level. For example, it is shown how algebra emerges from projective geometry, and how the hyperbolic plane emerges from the real projective line.

The author begins with Euclid-style construction and axiomatics, then proceeds to linear algebra when it becomes convenient to replace tortuous arguments with simple calculations. Next, he uses projective geometry to explain why objects look the way they do, as well as to explain why geometry is entangled with algebra. Lastly, the author introduces transformation groups—not only to clarify the differences between geometries, but also to exhibit geometries that are unexpectedly the same.

All readers are sure to find something new in this attractive text, which is abundantly supplemented with figures and exercises. This book will be useful for an undergraduate geometry course, a capstone course, or a course aimed at future high school teachers.

John Stillwell is Professor of Mathematics at the University of San Francisco. He is the author of several highly regarded books published by Springer, including *Elements of Number Theory* (2003), *Mathematics and Its History* (Second Edition, 2002), *Numbers and Geometry* (1998), and *Elements of Algebra* (1994).

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