

How can actuaries best equip themselves for the products and risk structures of the future? In this ground-breaking textbook, three leaders in actuarial science give a modern perspective on life contingencies.

The book begins with actuarial models and theory, emphasizing practical applications using computational techniques. The authors then develop a more contemporary outlook, introducing multiple state models, emerging cash flows and embedded options. This expanded edition contains more examples and exercises designed to help with exam preparation as well as developing up-to-date expertise. There are brand new sections and chapters on discrete time Markov processes, on models involving joint lives and on Universal Life insurance and participating traditional insurance.

Balancing rigour with intuition, and emphasizing applications, this textbook is ideal for university courses, for qualified actuaries wishing to renew and update their skills and for individuals preparing for the professional actuarial examinations of the Society of Actuaries or Institute and Faculty of Actuaries. The book covers the entire SOA MLC syllabus and will be especially valuable for students preparing for the new, long answer exam questions.

'The book is well written, well organized, and easy to read. It may be an excellent textbook for both undergraduate and graduate programmes in actuarial science. It is also a rich source of useful information for practitioners of the actuarial profession and financial risk managers who seek a practical and inspiring guide to liability cash flow modelling and valuation.'

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