

Contents

Preface

page xiii

1	Introduction	1
1.1	Means and Ends	1
1.2	The First Regression: A Historical Prelude	2
1.3	Quantiles, Ranks, and Optimization	5
1.4	Preview of Quantile Regression	9
1.5	Three Examples	13
1.5.1	Salaries versus Experience	13
1.5.2	Student Course Evaluations and Class Size	17
1.5.3	Infant Birth Weight	20
1.6	Conclusion	25
2	Fundamentals of Quantile Regression	26
2.1	Quantile Treatment Effects	26
2.2	How Does Quantile Regression Work?	32
2.2.1	Regression Quantiles Interpolate p Observations	33
2.2.2	The Subgradient Condition	34
2.2.3	Equivariance	38
2.2.4	Censoring	40
2.3	Robustness	42
2.3.1	The Influence Function	42
2.3.2	The Breakdown Point	45
2.4	Interpreting Quantile Regression Models	47
2.4.1	Some Examples	48
2.5	Caution: Quantile Crossing	55
2.6	A Random Coefficient Interpretation	59
2.7	Inequality Measures and Their Decomposition	62
2.8	Expectiles and Other Variations	63
2.9	Interpreting Misspecified Quantile Regressions	65
2.10	Problems	66

3	Inference for Quantile Regression	68
3.1	The Finite-Sample Distribution of Regression Quantiles	68
3.2	A Heuristic Introduction to Quantile Regression	
	Asymptotics	71
3.2.1	Confidence Intervals for the Sample Quantiles	72
3.2.2	Quantile Regression Asymptotics with IID Errors	73
3.2.3	Quantile Regression Asymptotics in Non-IID Settings	74
3.3	Wald Tests	75
3.3.1	Two-Sample Tests of Location Shift	75
3.3.2	General Linear Hypotheses	76
3.4	Estimation of Asymptotic Covariance Matrices	77
3.4.1	Scalar Sparsity Estimation	77
3.4.2	Covariance Matrix Estimation in Non-IID Settings	79
3.5	Rank-Based Inference	81
3.5.1	Rank Tests for Two-Sample Location Shift	81
3.5.2	Linear Rank Statistics	84
3.5.3	Asymptotics of Linear Rank Statistics	85
3.5.4	Rank Tests Based on Regression Rankscores	87
3.5.5	Confidence Intervals Based on Regression Rankscores	91
3.6	Quantile Likelihood Ratio Tests	92
3.7	Inference on the Quantile Regression Process	95
3.7.1	Wald Processes	97
3.7.2	Quantile Likelihood Ratio Processes	98
3.7.3	The Regression Rankscore Process Revisited	98
3.8	Tests of the Location-Scale Hypothesis	98
3.9	Resampling Methods and the Bootstrap	105
3.9.1	Bootstrap Refinements, Smoothing, and Subsampling	107
3.9.2	Resampling on the Subgradient Condition	108
3.10	Monte Carlo Comparison of Methods	110
3.10.1	Model 1: A Location-Shift Model	111
3.10.2	Model 2: A Location-Scale-Shift Model	112
3.11	Problems	113
4	Asymptotic Theory of Quantile Regression	116
4.1	Consistency	117
4.1.1	Univariate Sample Quantiles	117
4.1.2	Linear Quantile Regression	118
4.2	Rates of Convergence	120
4.3	Bahadur Representation	122
4.4	Nonlinear Quantile Regression	123
4.5	The Quantile Regression Rankscore Process	124
4.6	Quantile Regression Asymptotics under Dependent Conditions	126

4.6.1	Autoregression	126
4.6.2	ARMA Models	128
4.6.3	ARCH-like Models	129
4.7	Extremal Quantile Regression	130
4.8	The Method of Quantiles	131
4.9	Model Selection, Penalties, and Large- p Asymptotics	133
4.9.1	Model Selection	134
4.9.2	Penalty Methods	135
4.10	Asymptotics for Inference	138
4.10.1	Scalar Sparsity Estimation	139
4.10.2	Covariance Matrix Estimation	141
4.11	Resampling Schemes and the Bootstrap	141
4.12	Asymptotics for the Quantile Regression Process	142
4.12.1	The Durbin Problem	142
4.12.2	Khmaladization of the Parametric Empirical Process	144
4.12.3	The Parametric Quantile Process	145
4.12.4	The Parametric Quantile Regression Process	146
4.13	Problems	149
5	L-Statistics and Weighted Quantile Regression	151
5.1	L-Statistics for the Linear Model	151
5.1.1	Optimal L-Estimators of Location and Scale	153
5.1.2	L-Estimation for the Linear Model	155
5.2	Kernel Smoothing for Quantile Regression	158
5.2.1	Kernel Smoothing of the ρ_τ -Function	160
5.3	Weighted Quantile Regression	160
5.3.1	Weighted Linear Quantile Regression	160
5.3.2	Estimating Weights	161
5.4	Quantile Regression for Location-Scale Models	164
5.5	Weighted Sums of ρ_τ -Functions	168
5.6	Problems	170
6	Computational Aspects of Quantile Regression	173
6.1	Introduction to Linear Programming	173
6.1.1	Vertices	174
6.1.2	Directions of Descent	176
6.1.3	Conditions for Optimality	177
6.1.4	Complementary Slackness	178
6.1.5	Duality	180
6.2	Simplex Methods for Quantile Regression	181
6.3	Parametric Programming for Quantile Regression	185
6.3.1	Parametric Programming for Regression Rank Tests	188
6.4	Interior Point Methods for Canonical LPs	190
6.4.1	Newton to the Max: An Elementary Example	193
6.4.2	Interior Point Methods for Quantile Regression	199

6.4.3	Interior vs. Exterior: A Computational Comparison	202
6.4.4	Computational Complexity	204
6.5	Preprocessing for Quantile Regression	206
6.5.1	“Selecting” Univariate Quantiles	207
6.5.2	Implementation	207
6.5.3	Confidence Bands	208
6.5.4	Choosing m	209
6.6	Nonlinear Quantile Regression	211
6.7	Inequality Constraints	213
6.8	Weighted Sums of ρ_τ -Functions	214
6.9	Sparsity	216
6.10	Conclusion	220
6.11	Problems	220
7	Nonparametric Quantile Regression	222
7.1	Locally Polynomial Quantile Regression	222
7.1.1	Average Derivative Estimation	226
7.1.2	Additive Models	228
7.2	Penalty Methods for Univariate Smoothing	229
7.2.1	Univariate Roughness Penalties	229
7.2.2	Total Variation Roughness Penalties	230
7.3	Penalty Methods for Bivariate Smoothing	235
7.3.1	Bivariate Total Variation Roughness Penalties	235
7.3.2	Total Variation Penalties for Triograms	236
7.3.3	Penalized Triogram Estimation as a Linear Program	240
7.3.4	On Triangulation	241
7.3.5	On Sparsity	242
7.3.6	Automatic λ Selection	242
7.3.7	Boundary and Qualitative Constraints	243
7.3.8	A Model of Chicago Land Values	243
7.3.9	Taut Strings and Edge Detection	246
7.4	Additive Models and the Role of Sparsity	248
8	Twilight Zone of Quantile Regression	250
8.1	Quantile Regression for Survival Data	250
8.1.1	Quantile Functions or Hazard Functions?	252
8.1.2	Censoring	253
8.2	Discrete Response Models	255
8.2.1	Binary Response	255
8.2.2	Count Data	259
8.3	Quantile Autoregression	260
8.3.1	Quantile Autoregression and Comonotonicity	261
8.4	Copula Functions and Nonlinear Quantile Regression	265
8.4.1	Copula Functions	265

8.5	High-Breakdown Alternatives to Quantile Regression	268
8.6	Multivariate Quantiles	272
8.6.1	The Oja Median and Its Extensions	273
8.6.2	Half-Space Depth and Directional Quantile Regression	275
8.7	Penalty Methods for Longitudinal Data	276
8.7.1	Classical Random Effects as Penalized Least Squares	276
8.7.2	Quantile Regression with Penalized Fixed Effects	278
8.8	Causal Effects and Structural Models	281
8.8.1	Structural Equation Models	281
8.8.2	Chesher's Causal Chain Model	283
8.8.3	Interpretation of Structural Quantile Effects	284
8.8.4	Estimation and Inference	285
8.9	Choquet Utility, Risk, and Pessimistic Portfolios	287
8.9.1	Choquet Expected Utility	287
8.9.2	Choquet Risk Assessment	289
8.9.3	Pessimistic Portfolios	291
9	Conclusion	293
A	Quantile Regression in R: A Vignette	295
A.1	Introduction	295
A.2	What Is a Vignette?	296
A.3	Getting Started	296
A.4	Object Orientation	298
A.5	Formal Inference	299
A.6	More on Testing	305
A.7	Inference on the Quantile Regression Process	307
A.8	Nonlinear Quantile Regression	308
A.9	Nonparametric Quantile Regression	310
A.10	Conclusion	316
B	Asymptotic Critical Values	317
	<i>References</i>	319
	<i>Name Index</i>	337
	<i>Subject Index</i>	342