

Convex optimization problems arise frequently in many different fields. This book provides a comprehensive introduction to the subject, covering the theory, many applications and examples, and numerical methods. The book begins with the basic elements of convex sets and functions, describes various classes of convex optimization problems, and then treats duality theory. The second part covers a wide variety of applications, in estimation, approximation, statistics, computational geometry, and other areas. The last part of the book presents numerical methods for convex optimization problems, moving from basic methods for unconstrained problems to interior-point methods.

The focus of the book is on recognizing and formulating convex optimization problems, and then solving them efficiently. It contains many worked examples and homework exercises and will appeal to students, researchers, and practitioners in fields such as engineering, computer science, mathematics, finance, and economics.

Stephen Boyd received his Ph.D. from the University of California, Berkeley. Since 1985 he has been a member of the Electrical Engineering Department at Stanford University, where he is now the Samsung Professor of Engineering and Director of the Information Systems Laboratory. He has won numerous awards for research and teaching, and is a Fellow of the IEEE. He is the co-author of two previous books, *Linear Controller Design: Limits of Performance* and *Linear Matrix Inequalities in System and Control Theory*.

Lieven Vandenberghe received his Ph.D. from the Katholieke Universiteit Leuven, Belgium, and is Professor of Electrical Engineering at the University of California, Los Angeles. He has published widely in the field of optimization, and is co-editor of the *Handbook of Semidefinite Programming*.

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