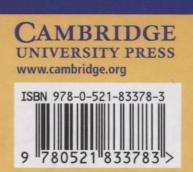
Convex optimization problems arise frequently in many different fields. This book provides a comprehensive introduction to the subject, covering the theory, many applications and examples, and numerical methods. The book begins with the basic elements of convex sets and functions, describes various classes of convex optimization problems, and then treats duality theory. The second part covers a wide variety of applications, in estimation, approximation, statistics, computational geometry, and other areas. The last part of the book presents numerical methods for convex optimization problems, moving from basic methods for unconstrained problems to interior-point methods.

The focus of the book is on recognizing and formulating convex optimization problems, and then solving them efficiently. It contains many worked examples and homework exercises and will appeal to students, researchers, and practitioners in fields such as engineering, computer science, mathematics, finance, and economics.

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