

**S**olving homework problems is the single most effective way for students to familiarize themselves with the language and details of solid state physics. Testing problem-solving ability is the best means at the professor's disposal for measuring student progress at critical points in the learning process. This book enables any instructor to supplement end-of-chapter textbook assignments with a large number of challenging and engaging practice problems and discover a host of new ideas for creating exam questions. In addition to numerous updates, this revised edition includes a fortified optics chapter and an entirely new chapter on mesoscopic and nanoscale physics.

Solid State Physics: Problems and Solutions is designed to be used in tandem with any of the excellent textbooks on this subject. Each problem has been chosen for its ability to illustrate key concepts, properties, and systems, knowledge of which is crucial in developing a complete understanding of the subject, including:

- Crystal Structures
- Interatomic Forces and Lattice Vibrations
- Electronic Band Structure
- Density of States
- Elementary Excitations
- Thermodynamics of Noninteracting Quasiparticles
- Optical Properties
- Interactions and Phase Transitions
- Mesoscopic and Nanoscale Systems



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