

Geometry, Analysis and Dynamics on sub-Riemannian Manifolds

Volume II

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Editors

Sub-Riemannian manifolds model media with constrained dynamics: motion at any point is only allowed along a limited set of directions, which are prescribed by the physical problem. From the theoretical point of view, sub-Riemannian geometry is the geometry underlying the theory of hypoelliptic operators and degenerate diffusions on manifolds.

In the last twenty years, sub-Riemannian geometry has emerged as an independent research domain, with extremely rich motivations and ramifications in several parts of pure and applied mathematics, such as geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics, optimal control and biology.

The aim of the lectures collected here is to present sub-Riemannian structures for the use of both researchers and graduate students.

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