

Geometry, Analysis and Dynamics on sub-Riemannian Manifolds

Volume I

Davide Barilari, Ugo Boscain and Mario Sigalotti
Editors

Sub-Riemannian manifolds model media with constrained dynamics: motion at any point is only allowed along a limited set of directions, which are prescribed by the physical problem. From the theoretical point of view, sub-Riemannian geometry is the geometry underlying the theory of hypoelliptic operators and degenerate diffusions on manifolds.

In the last twenty years, sub-Riemannian geometry has emerged as an independent research domain, with extremely rich motivations and ramifications in several parts of pure and applied mathematics, such as geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics, optimal control and biology.

The aim of the lectures collected here is to present sub-Riemannian structures for the use of both researchers and graduate students.

ISBN 978-3-03719-162-0



www.ems-ph.org

Contents

| | | |
|----|---|-----|
| 1 | Some topics of geometric measure theory in Carnot groups | 1 |
| | <i>Francesco Serra Cassano</i> | |
| 1 | Introduction | 1 |
| 2 | An introduction to Carnot groups | 3 |
| 3 | Differential calculus on Carnot groups | 20 |
| 4 | Differential calculus within Carnot groups | 34 |
| 5 | Sets of finite perimeter and minimal surfaces in Carnot groups | 83 |
| | Bibliography | 108 |
| 2 | Hypoelliptic operators and some aspects of analysis and geometry of sub-Riemannian spaces | 123 |
| | <i>Nicola Garofalo</i> | |
| 1 | Sub-Riemannian geometry and hypoelliptic operators | 123 |
| 2 | Carnot groups | 131 |
| 3 | Fundamental solutions and the Yamabe equation | 147 |
| 4 | Carnot–Carathéodory distance | 160 |
| 5 | Sobolev and BV spaces | 175 |
| 6 | Fractional integration in spaces of homogeneous type | 189 |
| 7 | Fundamental solutions of hypoelliptic operators | 202 |
| 8 | The geometric Sobolev embedding and the isoperimetric inequality | 212 |
| 9 | The Li–Yau inequality for complete manifolds with $\text{Ricci} \geq 0$ | 216 |
| 10 | Heat semigroup approach to the Li–Yau inequality | 224 |
| 11 | A heat equation approach to the volume doubling property | 233 |
| 12 | A sub-Riemannian curvature-dimension inequality | 239 |
| | Bibliography | 249 |
| 3 | Sub-Laplacians and hypoelliptic operators on totally geodesic Riemannian foliations | 259 |
| | <i>Fabrice Baudoin</i> | |
| 1 | Introduction | 259 |
| 2 | Riemannian foliations and their Laplacians | 261 |
| 3 | Horizontal Laplacians and heat kernels on model spaces | 268 |
| 4 | Transverse Weitzenböck formulas | 282 |
| 5 | The horizontal heat semigroup | 292 |
| 6 | The horizontal Bonnet–Myers theorem | 302 |
| 7 | Riemannian foliations and hypocoercivity | 308 |
| | Bibliography | 318 |
| | Index | 323 |