

# The area's benchmark text, completely revised and updated

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In the twenty years since publication of the first edition of *The Statistical Analysis of Failure Time Data*, researchers have produced a library of material on this constantly evolving area. The theoretical underpinnings of established methods have been strengthened, the scope of application has been extended, and counting process methods and related martingale convergence results have led to precise and general asymptotic results. Addressing graduate students, practitioners, and researchers, Jack Kalbfleisch and Ross Prentice update their classic text with these and other current developments in the second edition of *The Statistical Analysis of Failure Time Data*.

The authors include exercises and examples in each chapter, tying these sophisticated methods to practical applications. The *Second Edition* develops the dynamics of multivariate failure time data, extends the present material on Markov and semi Markov formulations, and includes an emphasis on left truncation. The final chapter on special topics and examples of data analysis has been completely revised and updated. Other chapters include:

- Inference in Parametric Models and Related Topics
- Relative Risk (Cox) Regression Models
- Competing Risks and Multistate Models
- Modeling and Analysis of Recurrent Event Data
- Analysis of Correlated Failure Time Data

With its comprehensive survey of the field and resources for students and researchers, *The Statistical Analysis of Failure Time Data* remains the benchmark text of the area.

**JOHN D. KALBFLEISCH, PhD**, is Professor of Biostatistics at the University of Michigan in Ann Arbor and the University of Waterloo in Ontario, Canada.

**ROSS L. PRENTICE, PhD**, is Professor of Biostatistics at the Fred Hutchinson Cancer Research Center and the University of Washington in Seattle.

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