

Contents

1	Jet Prolongations of Fibered Manifolds	1
1.1	The Rank Theorem	1
1.2	Fibered Manifolds	6
1.3	The Contact of Differentiable Mappings	10
1.4	Jet Prolongations of Fibered Manifolds	13
1.5	The Horizontalization	17
1.6	Jet Prolongations of Automorphisms of Fibered Manifolds	20
1.7	Jet Prolongations of Vector Fields	23
	References	33
2	Differential Forms on Jet Prolongations of Fibered Manifolds	35
2.1	The Contact Ideal	35
2.2	The Trace Decomposition	43
2.3	The Horizontalization	53
2.4	The Canonical Decomposition	58
2.5	Contact Components and Geometric Operations	67
2.6	Strongly Contact Forms	68
2.7	Fibered Homotopy Operators on Jet Prolongations of Fibered Manifolds	74
	References	84
3	Formal Divergence Equations	85
3.1	Formal Divergence Equations	85
3.2	Integrability of Formal Divergence Equations	89
3.3	Projectable Extensions of Differential Forms	93
	Reference	101

4	Variational Structures	103
4.1	Variational Structures on Fibered Manifolds	104
4.2	Variational Derivatives	108
4.3	Lepage Forms	112
4.4	Euler–Lagrange Forms	123
4.5	Lepage Equivalents and the Euler–Lagrange Mapping	124
4.6	The First Variation Formula	129
4.7	Extremals	130
4.8	Trivial Lagrangians	133
4.9	Source Forms and the Vainberg–Tonti Lagrangians	135
4.10	The Inverse Problem of the Calculus of Variations	146
4.11	Local Variationality of Second-Order Source Forms	157
	References	166
5	Invariant Variational Structures	169
5.1	Invariant Differential Forms	170
5.2	Invariant Lagrangians and Conservation Equations	172
5.3	Invariant Euler–Lagrange Forms	177
5.4	Symmetries of Extremals and Jacobi Vector Fields	178
	References	185
6	Examples: Natural Lagrange Structures	187
6.1	The Hilbert Variational Functional	188
6.2	Natural Lagrange Structures	194
6.3	Connections	197
	References	200
7	Elementary Sheaf Theory	201
7.1	Sheaf Spaces	201
7.2	Abelian Sheaf Spaces	207
7.3	Sections of Abelian Sheaf Spaces	211
7.4	Abelian Presheaves	213
7.5	Sheaf Spaces Associated with Abelian Presheaves	217
7.6	Sheaves Associated with Abelian Presheaves	221
7.7	Sequences of Abelian Groups, Complexes	226
7.8	Exact Sequences of Abelian Sheaves	238
7.9	Cohomology Groups of a Sheaf	242
7.10	Sheaves over Paracompact Hausdorff Spaces	250
	References	261

8	Variational Sequences	263
8.1	The Contact Sequence	264
8.2	The Variational Sequence	272
8.3	Variational Projectors	273
8.4	The Euler–Lagrange Morphisms	288
8.5	Variationally Trivial Lagrangians	296
8.6	Global Inverse Problem of the Calculus of Variations	298
	References.	300
	Appendix: Analysis on Euclidean Spaces and Smooth Manifolds	303
	Bibliography	341
	Index	347