Contents

Preface ix

Guide to the Book xvii

Chapter 1 Introduction

- 1.1 Overview and Notation
- 1.2 Lowpass Filter = Moving Average 7
- 1.3 Highpass Filter = Moving Difference 12
- 1.4 Filter Bank = Lowpass and Highpass 15
- 1.5 Scaling Function and Wavelets 22
- 1.6 Wavelet Transforms by Multiresolution 28

Chapter 2 Filters

- 2.1 Signals, Samples, and Time-Invariance 37
- 2.2 Ideal Filters, Shannon Sampling, Sinc Wavelets 45

1

- 2.3 Lowpass and Highpass Filter Design 53
- 2.4 Fourier Analysis 61
- 2.5 Bases and Frames 69
- 2.6 Time, Frequency, and Scale 80

Chapter 3 Downsampling and Upsampling

- 3.1 Matrices for Downsampling and Upsampling 87
- 3.2 Subsampling in the Frequency Domain 91
- 3.3 Sampling Operations in the z-Domain 96
- 3.4 Filters Interchanged with Samplers 100

Chapter 4 Filter Banks

- 4.1 Perfect Reconstruction 103
- 4.2 The Polyphase Matrix 114
- 4.3 Efficient Filter Banks 122
- 4.4 Polyphase for Upsampling and Reconstruction 128
- 4.5 Lattice Structure 134

Chapter 5 Orthogonal Filter Banks

- 5.1 Paraunitary Matrices 144
- 5.2 Orthonormal Filter Banks 147
- 5.3 Halfband Filters 153
- 5.4 Spectral Factorization 157
- 5.5 Maxflat (Daubechies) Filters 164

Chapter 6 Multiresolution

- 6.1 The Idea of Multiresolution 174
- 6.2 Wavelets from Filters 186
- 6.3 Computing the Scaling Function by Recursion 193
- 6.4 Infinite Product Formula 201
- 6.5 Biorthogonal Wavelets 208

Chapter 7 Wavelet Theory

- 7.1 Accuracy of Approximation 221
- 7.2 The Cascade Algorithm for the Dilation Equation 234
- 7.3 Smoothness of Scaling Functions and Wavelets 242
- 7.4 Splines and Semiorthogonal Wavelets 250
- 7.5 Multifilters and Multiwavelets 259

Chapter 8 Finite Length Signals

- 8.1 Circular Convolution and the DFT 263
- 8.2 Symmetric Extension for Symmetric Filters 272
- 8.3 Cosine Bases and the DCT 276
- 8.4 Smooth Local Cosine Bases 282
- 8.5 Boundary Filters and Wavelets 289

Chapter 9 M-Channel Filter Banks

- 9.1 Freedom versus Structure 299
- 9.2 Polyphase Form: M Channels 304
- 9.3 Perfect Reconstruction, Linear Phase, Orthogonality 316
- 9.4 Cosine-modulated Filter Banks 325
- 9.5 Multidimensional Filters and Wavelets 331

Contents

Chapter 10 Design Methods

10.1 **Distortions in Image Compression** 337 10.2 Design Methods — General Perspective 343 10.3 Design of Perfect Reconstruction Filter Banks 347 10.4 Design of Two-Channel Filter Banks 352 10.5 **Design of Cosine-modulated Filter Banks** 356 Chapter 11 Applications 11.1 Digitized Fingerprints and the FBI 362 11.2 Image and Video Compression 365 11.3 Speech, Audio, and ECG Compression 383 11.4 Shrinkage, Denoising, and Feature Detection 387 11.5 Communication Applications and Adaptive Systems 391 11.6 Wavelet Integrals for Differential Equations 394 Glossary 403 Appendix 1 Wavelets (American Scientist) 433 Appendix 2 Wavelets and Dilation Equations (SIAM Review) 440 MATLAB and the Wavelet Toolbox 453 References 475 Appendix 3 The Discrete Cosine Transform (SIAM Review) 487 Appendix 4 The Lifting Scheme 496 Block Transforms in Progressive Image Coding Appendix 5 501

Index 515