

Contents

| | |
|----------------------------------------------------|----|
| 1 Ordered sets; residuated mappings | 1 |
| 1.1 The concept of an order | 1 |
| 1.2 Order-preserving mappings | 5 |
| 1.3 Residuated mappings | 6 |
| 1.4 Closures | 10 |
| 1.5 Isomorphisms of ordered sets | 12 |
| 1.6 Galois connections | 14 |
| 1.7 Semigroups of residuated mappings | 15 |
| 2 Lattices; lattice morphisms | 19 |
| 2.1 Semilattices and lattices | 19 |
| 2.2 Down-set lattices | 23 |
| 2.3 Sublattices | 26 |
| 2.4 Lattice morphisms | 28 |
| 2.5 Complete lattices | 29 |
| 2.6 Baer semigroups | 35 |
| 3 Regular equivalences | 39 |
| 3.1 Ordering quotient sets | 39 |
| 3.2 Strongly upper regular equivalences | 41 |
| 3.3 Lattice congruences | 45 |
| 4 Modular lattices | 49 |
| 4.1 Modular pairs; Dedekind's modularity criterion | 49 |
| 4.2 Chain conditions | 54 |
| 4.3 Join-irreducibles | 58 |
| 4.4 Baer semigroups and modularity | 61 |
| 5 Distributive lattices | 65 |
| 5.1 Birkhoff's distributivity criterion | 65 |
| 5.2 More on join-irreducibles | 69 |
| 5.3 Prime ideals and filters | 72 |
| 5.4 Baer semigroups and distributivity | 74 |

| | |
|------------------------------------------------------------|-----|
| 6 Complementation; boolean algebras | 77 |
| 6.1 Complemented elements | 77 |
| 6.2 Uniquely complemented lattices | 78 |
| 6.3 Boolean algebras and boolean rings | 82 |
| 6.4 Boolean algebras of subsets | 86 |
| 6.5 The Dedekind–MacNeille completion of a boolean algebra | 90 |
| 6.6 Neutral and central elements | 92 |
| 6.7 Stone’s representation theorem | 95 |
| 6.8 Baer semigroups and complementation | 97 |
| 6.9 Generalisations of boolean algebras | 101 |
| 7 Pseudocomplementation; Stone and Heyting algebras | 103 |
| 7.1 Pseudocomplements | 103 |
| 7.2 Stone algebras | 106 |
| 7.3 Heyting algebras | 111 |
| 7.4 Baer semigroups and residuation | 116 |
| 8 Congruences; subdirectly irreducible algebras | 119 |
| 8.1 More on lattice congruences | 119 |
| 8.2 Congruence kernels | 121 |
| 8.3 Principal congruences | 126 |
| 8.4 Congruences on p -algebras | 130 |
| 8.5 Congruences on Heyting algebras | 134 |
| 8.6 Subdirectly irreducible algebras | 137 |
| 9 Ordered groups | 143 |
| 9.1 Ordering groups | 143 |
| 9.2 Convex subgroups | 147 |
| 9.3 Lattice-ordered groups | 150 |
| 9.4 Absolute values and orthogonality | 153 |
| 9.5 Convex ℓ -subgroups | 158 |
| 9.6 Polars | 162 |
| 9.7 Coset ordering; prime subgroups | 164 |
| 9.8 Representable groups | 168 |
| 10 Archimedean ordered structures | 171 |
| 10.1 Totally ordered rings and fields | 171 |
| 10.2 Archimedean ordered fields | 177 |
| 10.3 Archimedean totally ordered groups | 188 |
| 11 Ordered semigroups; residuated semigroups | 193 |
| 11.1 Ordered semigroups | 193 |
| 11.2 Residuated semigroups | 197 |
| 11.3 Molinaro equivalences | 204 |

| | |
|-------------------------------------------------------------------------|-----|
| 12 Epimorphic group images; Dubreil-Jacotin semigroups | 207 |
| 12.1 Anticones | 207 |
| 12.2 Dubreil-Jacotin semigroups | 212 |
| 12.3 Residuated Dubreil-Jacotin semigroups | 217 |
| 13 Ordered regular semigroups | 225 |
| 13.1 Regular Dubreil-Jacotin semigroups | 225 |
| 13.2 The Nambooripad order | 228 |
| 13.3 Natural orders on regular semigroups | 232 |
| 13.4 Biggest inverses | 243 |
| 13.5 Principally ordered regular semigroups | 251 |
| 13.6 Principally and naturally ordered semigroups | 255 |
| 13.7 Ordered completely simple semigroups | 258 |
| 14 Structure theorems | 265 |
| 14.1 Naturally ordered regular semigroups | 265 |
| 14.1.1 Inverse transversals | 265 |
| 14.1.2 Biggest idempotent | 269 |
| 14.1.3 Biggest inverses | 269 |
| 14.2 Integral Dubreil-Jacotin inverse semigroups | 271 |
| 14.3 Orthodox Dubreil-Jacotin semigroups | 274 |
| 14.3.1 The cartesian order | 276 |
| 14.3.2 Unilateral lexicographic orders | 282 |
| 14.3.3 Bootlace orders | 285 |
| 14.3.4 Lexicographic orders | 290 |
| 14.4 Lattices for which $\text{Res } L$ is regular | 291 |
| References | 293 |
| Index | 299 |