

Lecture Notes in Physics 835

Martin Bojowald

Quantum Cosmology

A Fundamental Description of the Universe

The universe, ultimately, is to be described by quantum theory. Quantum aspects of all there is, including space and time, may not be significant for many purposes, but are crucial for some time. And so a quantum description of cosmology is required for a complete and consistent worldview. Consequences of quantum gravity on grander scales are expected to be enormous. In Quantum Cosmology, A Fundamental Description of the Universe, Martin Bojowald discusses his theory to see how black holes behave and where our universe came from. Applications like loop quantum gravity and cosmology have by now shed much light on cosmic evolution of a universe in a fundamental, microscopic description. Modern techniques demonstrate how the universe may have come from a non-singular phase before the Big Bang, how equations for the evolution of structure can be derived, how observations could be used to test these claims, but also what fundamental limitations remain to our knowledge of the universe before the Big Bang.

Physics

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