Contents

Preface ix

Chapter 1 First-Order Differential Equations 1
- 1.1 Basic Results 1
- 1.2 First-Order Linear Equations 4
- 1.3 Autonomous Equations 5
- 1.4 Generalized Logistic Equation 10
- 1.5 Bifurcation 14
- 1.6 Exercises 16

Chapter 2 Linear Systems 23
- 2.1 Introduction 23
- 2.2 The Vector Equation $x' = A(t)x$ 27
- 2.3 The Matrix Exponential Function 42
- 2.4 Induced Matrix Norm 59
- 2.5 Floquet Theory 64
- 2.6 Exercises 76

Chapter 3 Autonomous Systems 87
- 3.1 Introduction 87
- 3.2 Phase Plane Diagrams 90
- 3.3 Phase Plane Diagrams for Linear Systems 96
- 3.4 Stability of Nonlinear Systems 107
- 3.5 Linearization of Nonlinear Systems 113
- 3.6 Existence and Nonexistence of Periodic Solutions 120
- 3.7 Three-Dimensional Systems 134
- 3.8 Differential Equations and *Mathematica* 145
- 3.9 Exercises 149

Chapter 4 Perturbation Methods 161
- 4.1 Introduction 161
- 4.2 Periodic Solutions 172
- 4.3 Singular Perturbations 178
- 4.4 Exercises 186

Chapter 5 The Self-Adjoint Second-Order Differential Equation 192
- 5.1 Basic Definitions 192

5.2 An Interesting Example 197
5.3 Cauchy Function and Variation of Constants Formula 199
5.4 Sturm-Liouville Problems 204
5.5 Zeros of Solutions and Disconjugacy 212
5.6 Factorizations and Recessive and Dominant Solutions 219
5.7 The Riccati Equation 229
5.8 Calculus of Variations 240
5.9 Green's Functions 251
5.10 Exercises 272

Chapter 6 Linear Differential Equations of Order n 281
6.1 Basic Results 281
6.2 Variation of Constants Formula 283
6.3 Green's Functions 287
6.4 Factorizations and Principal Solutions 297
6.5 Adjoint Equation 302
6.6 Exercises 307

Chapter 7 BVPs for Nonlinear Second-Order DEs 309
7.1 Contraction Mapping Theorem (CMT) 309
7.2 Application of the CMT to a Forced Equation 311
7.3 Applications of the CMT to BVPs 313
7.4 Lower and Upper Solutions 325
7.5 Nagumo Condition 334
7.6 Exercises 340

Chapter 8 Existence and Uniqueness Theorems 345
8.1 Basic Results 345
8.2 Lipschitz Condition and Picard-Lindelof Theorem 348
8.3 Equicontinuity and the Ascoli-Arzela Theorem 356
8.4 Cauchy-Peano Theorem 358
8.5 Extendability of Solutions 363
8.6 Basic Convergence Theorem 369
8.7 Continuity of Solutions with Respect to ICs 372
8.8 Kneser's Theorem 375
8.9 Differentiating Solutions with Respect to ICs 378
8.10 Maximum and Minimum Solutions 387
8.11 Exercises 396

Solutions to Selected Problems 403

Bibliography 415

Index 419