

Contents

PART III. SPECTRAL OPERATORS

XV. Spectral Operators	1925
1. Introduction	1925
2. Terminology and Preliminary Notions	1928
3. The Resolvent of a Spectral Operator	1933
4. The Canonical Reduction of a Spectral Operator	1937
5. An Operational Calculus for Bounded Spectral Operators	1941
6. Bounded Spectral Operators in Hilbert Space	1945
7. Relations Between a Spectral Operator and Its Scalar Part	1950
8. The Spectrum of a Spectral Operator	1954
9. The Algebras $\mathfrak{U}^p$ and $\mathfrak{U}^p$	1959
10. The Spectral Analysis of Operators in $\mathfrak{U}^p$	1970
11. Some Examples of Bounded Spectral Operators	1983
12. Some Examples of Unbounded Spectral Operators	2010
13. Determinism	2054
14. Wiener-Lévy-Hopf Theorems	2063
15. Exercises	2074
16. Notes and Remarks	2089
XVI. Spectral Operators: Sufficient Conditions	2134
1. Statement of the Problem	2134
2. Consequences of the Condition (A)	2136
3. Consequences of the Conditions (A) and (B)	2138
4. Consequences of the Conditions (A, B, C): Necessary and Sufficient Conditions for Spectral Operators	2145
5. Operators Whose Spectra Lie in a Jordan Curve	2149
6. Self Adjoint Operators in Hilbert Space	2169
7. Exercises	2171
8. Notes and Remarks	2173

XVII.	Algebras of Spectral Operators	2177
1.	Introduction	2177
2.	The Structure of Commutative B -Algebras of Spectral Operators	2179
3.	Strongly Closed Algebras and Complete Boolean Algebras	2194
4.	Strong Limits of Spectral Operators: Non-Commutative Case	2219
5.	Exercises	2222
6.	Notes and Remarks	2225
XVIII.	Unbounded Spectral Operators	2227
1.	Introduction	2227
2.	Unbounded Spectral Operators	2228
3.	Multiplicity Theory and Spectral Representation	2264
4.	Notes and Remarks,	2288
XIX.	Perturbations of Spectral Operators with Discrete Spectra	2290
1.	Introduction	2290
2.	The Principal Abstract Perturbation Theorem	2291
3.	Separated Boundary Conditions for the Second Order Operator	2307
4.	Spectral Properties of $[(1/i)(d/dx)]^n$	2319
5.	Approximation by Generalized Eigenfunctions	2351
6.	Notes and Remarks	2371
XX.	Spectral Operators with Continuous Spectra: Applications of the General Theory	2375
1.	Spectral Differential Operators of Second Order	2378
2.	Friedrichs' Method of Similar Operators	2400
3.	The Friedrichs' Method for the Discrete Spectrum	2448
4.	The Wave Operator Method	2453
5.	Exercises	2480
6.	Notes and Remarks	2490

17
CONTENTS

xix

References		2512
Notation Index		2577
Author Index		2579
Subject Index		2587