# WILEY-VCH

"A Modern Course in Statistical Physics" is a textbook that illustrates the foundations of equilibrium and non-equilibrium statistical physics, and the universal nature of thermodynamic processes, from the point of view of contemporary research problems.

The book treats such diverse topics as the microscopic theory of critical phenomena, superfluid dynamics, quantum conductance, light scattering, transport processes, and dissipative structures, all in the framework of the foundations of statistical physics and thermodynamics. It shows the quantum origins of problems in classical statistical physics. One focus of the book is fluctuations that occur due to the discrete nature of matter, a topic of growing importance for nanometer scale physics and biophysics. Another focus concerns classical and quantum phase transitions, in both monatomic and mixed particle systems.

This fourth Edition extends the range of topics considered to include, for example, entropic forces, electrochemical processes in biological systems and batteries, adsorption processes in biological systems, diamagnetism, the theory of Bose-Einstein condensation, memory effects in Brownian motion, the hydrodynamics of binary mixtures.

A set of exercises and problems is to be found at the end of each chapter and, in addition, solutions to a subset of the problems is provided. The appendices cover Exact Differentials, Ergodicity, Number Representation, Scattering Theory, and also a short course on Probability.



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