

CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS

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This highly acclaimed series of monographs provides introductory accounts of specialized topics in mathematical physics for graduate students and research workers. The monographs in this series are of outstanding scholarship and written by those at the very frontiers of research. Subject areas covered include cosmology, astrophysics, relativity theory, particle physics, quantum theory, nuclear physics, statistical mechanics, condensed matter physics, plasma physics and the theory of chaos.

ABOUT THIS BOOK

Quantum field theory in curved spacetime has been remarkably fruitful. It can be used to explain how the large-scale structure of the universe and the anisotropies of the cosmic background radiation that we observe today first arose. Similarly, it provides a deep connection between general relativity, thermodynamics, and quantum field theory. This book develops quantum field theory in curved spacetime in a pedagogical style, suitable for graduate students.

The authors present detailed, physically motivated derivations of cosmological and black hole processes in which curved spacetime plays a key role. They explain how such processes in the rapidly expanding early universe leave observable consequences today, and how in the context of evaporating black holes, these processes uncover deep connections between gravitation and elementary particles. The authors also lucidly describe many other aspects of free and interacting quantized fields in curved spacetime.

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