

**CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS**

Edited by

P.V. LANDSHOFF *Professor of Mathematical Physics, University of Cambridge*

D.R. NELSON *Professor of Physics, Harvard University*

S. WEINBERG *Josey Regental Professor of Science, University of Texas at Austin*

This highly acclaimed series of monographs provides introductory accounts of specialized topics in mathematical physics for graduate students and research workers. The monographs in this series are of outstanding scholarship and written by those at the very frontiers of research. Subject areas covered include cosmology, astrophysics, relativity theory, particle physics, quantum theory, nuclear physics, statistical mechanics, condensed matter physics, plasma physics and the theory of chaos.

**ABOUT THIS BOOK**

Quantum field theory in curved spacetime has been remarkably fruitful. It can be used to explain how the large-scale structure of the universe and the anisotropies of the cosmic background radiation that we observe today first arose. Similarly, it provides a deep connection between general relativity, thermodynamics, and quantum field theory. This book develops quantum field theory in curved spacetime in a pedagogical style, suitable for graduate students.

The authors present detailed, physically motivated derivations of cosmological and black hole processes in which curved spacetime plays a key role. They explain how such processes in the rapidly expanding early universe leave observable consequences today, and how in the context of evaporating black holes, these processes uncover deep connections between gravitation and elementary particles. The authors also lucidly describe many other aspects of free and interacting quantized fields in curved spacetime.

**CAMBRIDGE**  
UNIVERSITY PRESS  
[www.cambridge.org](http://www.cambridge.org)

ISBN 978-0-521-87787-9



9 780521 877879 >



# Contents

<i>Preface</i>	page xi
<i>Acknowledgments</i>	xiii
<i>Conventions and notation</i>	xv
<b>1 Quantum fields in Minkowski spacetime</b>	<b>1</b>
1.1 Canonical formulation	2
1.2 Particles	13
1.3 Vacuum energy	21
1.4 Charged scalar field	24
1.5 Dirac field	27
1.6 Angular momentum and spin	32
<b>2 Basics of quantum fields in curved spacetimes</b>	<b>36</b>
2.1 Canonical quantization and conservation laws	37
2.2 Scalar field	43
2.3 Cosmological model: Arbitrary asymptotically static time dependence	47
2.4 Particle creation in a dynamic universe	51
2.5 Statistics from dynamics: Spin-0	54
2.6 Conformally invariant non-interacting field	56
2.7 Probability distribution of created particles	58
2.8 Exact solution with particle creation	61
2.9 High-frequency blackbody distribution	63
2.10 de Sitter spacetime	64
2.11 Quantum fluctuations and early inflation	73
2.12 Quantizing the inflaton field perturbations	77
2.13 A word on interacting quantized fields and on algebraic quantum field theory in curved spacetime	88
2.14 Accelerated detector in Minkowski spacetime	91
<b>3 Expectation values quadratic in fields</b>	<b>93</b>
3.1 Adiabatic subtraction and physical quantities	93
3.2 Energy-momentum tensor from trace anomaly	107
3.3 Renormalization in general spacetimes	110
3.4 Gaussian approximation to propagator	115



3.5	Approximate energy-momentum tensor in Schwarzschild, de Sitter, and other static Einstein spacetimes	118
3.6	$R$ -summed form of propagator	129
3.7	$R$ -summed action and cosmic acceleration	131
3.8	Normal coordinate momentum space	134
3.9	Chiral current anomaly caused by spacetime curvature	144
<b>4</b>	<b>Particle creation by black holes</b>	<b>152</b>
4.1	Introduction	152
4.2	Classical considerations	153
4.3	Quantum aspects	162
4.4	Energy-momentum tensor with Hawking flux	174
4.5	Back reaction to black hole evaporation	178
4.6	Trans-Planckian physics in Hawking radiation and cosmology	180
4.7	Further topics: Closed timelike curves; closed-time-path integral	182
<b>5</b>	<b>The one-loop effective action</b>	<b>184</b>
5.1	Introduction	184
5.2	Preliminary definition of the effective action	185
5.3	Regularization of the effective action	190
5.4	Effective action for scalar fields: Some examples	200
5.5	The conformal anomaly and the functional integral	216
5.6	Spinors in curved spacetime	221
5.7	The effective action for spinor fields	245
5.8	Application of the effective action for spinor fields	253
5.9	The axial, or chiral, anomaly	257
<b>6</b>	<b>The effective action: Non-gauge theories</b>	<b>268</b>
6.1	Introduction	268
6.2	The Schwinger action principle	271
6.3	The Feynman path integral	277
6.4	The effective action	280
6.5	The geometrical effective action	283
6.6	Perturbative expansion of the effective action	295
6.7	Renormalization of an interacting scalar field theory	302
6.8	The renormalization group and the effective action	318
6.9	The effective potential	322
6.10	The renormalization of the non-linear sigma model	331
6.11	Formal properties of the effective action	337
	Appendix	344



<b>7 The effective action: Gauge theories</b>	<b>348</b>
7.1 Introduction	348
7.2 Gauge transformations	350
7.3 The orbit space and the gauge condition	359
7.4 Field space reparameterization and the Killing equation	368
7.5 The connection and its consequences	375
7.6 The functional measure for gauge theories	385
7.7 Gauge-invariant effective action	390
7.8 Yang–Mills theory, concluded	399
7.9 Scalar quantum electrodynamics	407
Appendix	420
<b>Appendix: Quantized Inflaton Perturbations</b>	<b>422</b>
<i>References</i>	426
<i>Index</i>	445