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Modern statistical methods use models that require the computation of probabilities from complicated distributions, which can lead to intractable computations. Saddlepoint approximations can be the answer. Written from the user's point of view, this book explains in clear, simple language how such approximate probability computations are made, taking readers from the very beginnings to current applications.

The book aims to make the subject accessible to the widest possible audience by using graduated levels of difficulty in which the core material is presented in Chapters 1–6 at an elementary mathematical level. Readers are guided in applying the methods in various computations that will build their skills and deepen their understanding when later complemented with discussion of theoretical aspects. Chapters 7–9 address the p^* and r^* formulas of higher-order asymptotic inference, developed through the Danish approach to the subject by Barndorff-Nielsen and others. These provide a readable summary of the literature and an overview of the subject beginning with the original work of Fisher. Later chapters address special topics where saddlepoint methods have had substantial impact through particular applications. These include applications in multivariate testing, applications to stochastic systems and applied probability, bootstrap implementation in the transform domain, and Bayesian computation and inference.

No previous background in the area is required, as the book introduces the subject from the very beginning. Many data examples from real applications show the methods at work and demonstrate their practical value. Ideal for graduate students and researchers in statistics, biostatistics, electrical engineering, econometrics, applied mathematics, and other fields where statistical and probabilistic modeling are used; this is both an entry-level text and a valuable reference.

RONALD W. BUTLER is Professor of Statistical Science and the Charles F. Frensley Chair in Mathematical Sciences at Southern Methodist University.

Cambridge Series in Statistical and Probabilistic Mathematics

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This series of high quality upper-division textbooks and expository monographs covers all aspects of stochastic applicable mathematics. The topics range from pure and applied statistics to probability theory, operations research, optimization and mathematical programming. The books contain clear presentations of new developments in the field and also of the state of the art in classical methods. While emphasizing rigorous treatment of theoretical methods, the books also contain applications and discussions of new techniques made possible by advances in computational practice.

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| | | |
|----------|--|------------|
| 1 | Fundamental approximations | 1 |
| 1.1 | Univariate densities and mass functions | 1 |
| 1.2 | Univariate cumulative distribution functions | 12 |
| 1.3 | Failure (hazard) rate approximation | 28 |
| 1.4 | Final remarks | 30 |
| 1.5 | Computational notes | 30 |
| 1.6 | Exercises | 31 |
| 2 | Properties and derivations | 38 |
| 2.1 | Simple properties of the approximations | 38 |
| 2.2 | Saddlepoint density | 41 |
| 2.3 | Saddlepoint CDF approximation | 49 |
| 2.4 | Further topics | 54 |
| 2.5 | Appendix | 66 |
| 2.6 | Exercises | 70 |
| 3 | Multivariate densities | 75 |
| 3.1 | Saddlepoint density and mass functions | 75 |
| 3.2 | Development of the saddlepoint density | 83 |
| 3.3 | Properties of multivariate saddlepoint densities | 91 |
| 3.4 | Further examples | 93 |
| 3.5 | Multivariate CDFs | 101 |
| 3.6 | Exercises | 102 |
| 4 | Conditional densities and distribution functions | 107 |
| 4.1 | Conditional saddlepoint density and mass functions | 107 |
| 4.2 | Conditional cumulative distribution functions | 113 |
| 4.3 | Further examples: Linear combinations of independent variables | 123 |
| 4.4 | Further topics | 126 |
| 4.5 | Appendix | 132 |
| 4.6 | Exercises | 136 |

| | | |
|-----------|--|------------|
| 5 | Exponential families and tilted distributions | 145 |
| 5.1 | Regular exponential families | 145 |
| 5.2 | Edgeworth expansions | 151 |
| 5.3 | Tilted exponential families and saddlepoint approximations | 156 |
| 5.4 | Saddlepoint approximation in regular exponential families | 158 |
| 5.5 | Exercises | 179 |
| 6 | Further exponential family examples and theory | 183 |
| 6.1 | Logistic regression and LD50 estimation | 183 |
| 6.2 | Common odds ratio in 2×2 tables | 193 |
| 6.3 | Times series analysis of truncated count data | 208 |
| 6.4 | Exponential families of Markov processes | 209 |
| 6.5 | Truncation | 212 |
| 6.6 | Exercises | 213 |
| 7 | Probability computation with p^* | 219 |
| 7.1 | The p^* density in regular exponential families | 219 |
| 7.2 | Conditional inference and p^* in group transformation models | 225 |
| 7.3 | Approximate conditional inference and p^* in curved exponential families | 230 |
| 7.4 | Appendix | 250 |
| 7.5 | Exercises | 254 |
| 8 | Probabilities with r^*-type approximations | 259 |
| 8.1 | Notation, models, and sample space derivatives | 259 |
| 8.2 | Scalar parameter approximations | 260 |
| 8.3 | Examples | 261 |
| 8.4 | Derivation of (8.1) | 265 |
| 8.5 | Other versions of \hat{u} | 266 |
| 8.6 | Numerical examples | 271 |
| 8.7 | Properties | 278 |
| 8.8 | Appendix | 279 |
| 8.9 | Exercises | 282 |
| 9 | Nuisance parameters | 285 |
| 9.1 | Approximation with nuisance parameters | 285 |
| 9.2 | Examples | 286 |
| 9.3 | Derivation of (9.3) and (9.4) | 291 |
| 9.4 | Exact and approximate sample space derivatives | 292 |
| 9.5 | Numerical examples | 294 |
| 9.6 | Variation independence, conditional likelihood, and marginal likelihood | 297 |
| 9.7 | Examples | 304 |
| 9.8 | Properties of (9.3) | 314 |
| 9.9 | Exercises | 315 |
| 10 | Sequential saddlepoint applications | 323 |
| 10.1 | Sequential saddlepoint approximation | 323 |

| | | |
|-----------|--|-----|
| 10.2 | Comparison to the double-saddlepoint approach | 324 |
| 10.3 | Examples | 325 |
| 10.4 | P -values for the Bartlett–Nanda–Pillai trace statistic | 330 |
| 10.5 | Exercises | 334 |
| 11 | Applications to multivariate testing | 341 |
| 11.1 | P -values in MANOVA | 342 |
| 11.2 | P -values in tests of covariance | 348 |
| 11.3 | Power functions for multivariate tests | 355 |
| 11.4 | Some multivariate saddlepoint densities | 363 |
| 11.5 | Appendix | 365 |
| 11.6 | Exercises | 366 |
| 12 | Ratios and roots of estimating equations | 374 |
| 12.1 | Ratios | 375 |
| 12.2 | Univariate roots of estimating equations | 384 |
| 12.3 | Distributions for vector ratios | 392 |
| 12.4 | Multivariate roots of estimating equations | 401 |
| 12.5 | The conditional CDF of R_m given R_1, \dots, R_{m-1} | 411 |
| 12.6 | Appendix | 420 |
| 12.7 | Exercises | 422 |
| 13 | First passage and time to event distributions | 430 |
| 13.1 | Semi-Markov and Markov processes with finite state space | 430 |
| 13.2 | Passage times with a single destination state | 435 |
| 13.3 | Passage times with many possible destination states | 452 |
| 13.4 | Birth and death processes and modular systems | 454 |
| 13.5 | Markov processes | 461 |
| 13.6 | A redundant and repairable system | 462 |
| 13.7 | Appendix | 466 |
| 13.8 | Exercises | 469 |
| 14 | Bootstrapping in the transform domain | 474 |
| 14.1 | Saddlepoint approximation to the single bootstrap distribution | 475 |
| 14.2 | Saddlepoint approximations for double bootstrap confidence bands | 482 |
| 14.3 | Semiparametric bootstrap | 487 |
| 14.4 | Indirect saddlepoint approximation | 494 |
| 14.5 | Empirical saddlepoint approximations | 500 |
| 14.6 | Appendix | 500 |
| 14.7 | Exercises | 502 |
| 15 | Bayesian applications | 506 |
| 15.1 | Bayesian prediction with intractable predictand distribution | 507 |
| 15.2 | Passage time examples for Markov processes | 510 |
| 15.3 | Passage time examples for semi-Markov processes | 517 |

| | | |
|-----------|---|-----|
| 15.4 | Conclusions, applicability, and alternative methods | 522 |
| 15.5 | Computational algorithms | 524 |
| 15.6 | Exercises | 525 |
| 16 | Nonnormal bases | 528 |
| 16.1 | Nonnormal-based saddlepoint expressions | 528 |
| 16.2 | Choice of base distribution | 532 |
| 16.3 | Conditional distribution approximation | 538 |
| 16.4 | Examples | 539 |
| 16.5 | Exercises | 545 |
| | <i>References</i> | 548 |
| | <i>Index</i> | 560 |