Modern statistical methods use models that require the computation of probabilities from complicated distributions, which can lead to intractable computations. Saddlepoint approximations can be the answer. Written from the user's point of view, this book explains in clear, simple language how such approximate probability computations are made, taking readers from the very beginnings to current applications.

The book aims to make the subject accessible to the widest possible audience by using graduated levels of difficulty in which the core material is presented in Chapters 1– 6 at an elementary mathematical level. Readers are guided in applying the methods in various computations that will build their skills and deepen their understanding when later complemented with discussion of theoretical aspects. Chapters 7– 9 address the  $p^*$  and  $r^*$  formulas of higherorder asymptotic inference, developed through the Danish approach to the subject by Barndorff-Nielsen and others. These provide a readable summary of the literature and an overview of the subject beginning with the original work of Fisher. Later chapters address special topics where saddlepoint methods have had substantial impact through particular applications. These include applications in multivariate testing, applications to stochastic systems and applied probability, bootstrap implementation in the transform domain, and Bayesian computation and inference.

No previous background in the area is required, as the book introduces the subject from the very beginning. Many data examples from real applications show the methods at work and demonstrate their practical value. Ideal for graduate students and researchers in statistics, biostatistics, electrical engineering, econometrics, applied mathematics, and other fields where statistical and probabilistic modeling are used; this is both an entry-level text and a valuable reference.

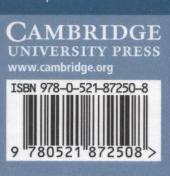
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## **Cambridge Series in Statistical and Probabilistic Mathematics**

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This series of high quality upper-division textbooks and expository monographs covers all aspects of stochastic applicable mathematics. The topics range from pure and applied statistics to probability theory, operations research, optimization and mathematical programming. The books contain clear presentations of new developments in the field and also of the state of the art in classical methods. While emphasizing rigorous treatment of theoretical methods, the books also contain applications and discussions of new techniques made possible by advances in computational practice.



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