

Contents

Preface	ix
Notes for the Reader	xi
1 Aims and Scope of This Book	1
2 Prehistory, Emergence, and Evolution of Fuzzy Logic	5
2.1 Prehistory of fuzzy logic	5
2.2 Emergence of fuzzy logic	17
2.3 Evolving attitudes toward fuzzy logic	27
2.4 Documented debates	32
2.5 Evolution of supporting infrastructure for fuzzy logic	38
3 Fuzzy Logic in the Broad Sense	43
3.1 Introduction	43
3.2 Basic concepts of fuzzy sets	44
3.3 Operations on fuzzy sets	46
3.4 Fuzzy intervals, fuzzy numbers, and fuzzy arithmetic	55
3.5 Fuzzy relations	60
3.6 Approximate reasoning	69
3.7 Possibility theory	79
3.8 Fuzzy clustering	88
3.9 Methods for constructing fuzzy sets	94
3.10 Nonstandard fuzzy sets	99
4 Fuzzy Logic in the Narrow Sense	105
4.1 From classical to fuzzy logic: Principal issues	106
4.2 Many-valued logics until the 1960s	112
4.2.1 Łukasiewicz finitely- and infinitely-valued logics	112
4.2.2 Gödel finitely- and infinitely-valued logics	129
4.2.3 Other propositional logics	134

4.2.4	Predicate logics	147
4.2.5	Further developments	155
4.2.6	The mid-1960s and Zadeh's idea of fuzzy sets	159
4.3	Fuzzy logics with graded consequence	160
4.3.1	Goguen's logic of inexact concepts	161
4.3.2	Pavelka-style fuzzy logic	165
4.3.3	Further developments	171
4.4	Fuzzy logics based on t-norms and their residua	177
4.4.1	Fuzzy logics based on t-norms until the mid-1990s	178
4.4.2	Product logic	187
4.4.3	Hájek's logic BL and <i>Metamathematics of Fuzzy Logic</i> . . .	189
4.4.4	Logics related to BL	202
4.5	Fuzzy logic and uncertainty	210
4.5.1	Degrees of truth vs. belief and truth functionality	210
4.5.2	Possibilistic logic	212
4.5.3	Gerla's probabilistic fuzzy logics	215
4.5.4	Belief, modality, and quantifiers in fuzzy logic	216
4.6	Miscellaneous issues	222
4.6.1	Relationship to applications	222
4.6.2	Computability and complexity	224
4.6.3	Further developments	227
5	Mathematics Based on Fuzzy Logic	231
5.1	Principal issues and outline of development	231
5.1.1	What is mathematics based on fuzzy logic?	231
5.1.2	The problem and role of foundations	233
5.1.3	The problem and role of applications	238
5.1.4	Outline of development	241
5.2	Foundations of mathematics based on fuzzy logic	246
5.2.1	The role of fuzzy logic in the narrow sense	246
5.2.2	Higher-order logic approaches	250
5.2.3	Set-theoretic approaches	253
5.2.4	Category-theoretic approaches	261
5.3	Selected areas of mathematics based on fuzzy logic	267
5.3.1	Sets and relations	267
5.3.2	Algebra	282
5.3.3	Topology	292
5.3.4	Quantities and mathematical analysis	298
5.3.5	Probability and statistics	309
5.3.6	Geometry	315
5.3.7	Further developments	320

5.4	Miscellaneous issues	321
5.4.1	Interpretation of truth degrees	321
5.4.2	Fuzzy logic and paradoxes	332
5.4.3	Fuzzy logic and vagueness	339
6	Applications of Fuzzy Logic	347
6.1	Introduction	347
6.2	A historical overview	349
6.3	Engineering	356
6.3.1	Fuzzy control	356
6.3.2	Other engineering applications	365
6.4	Decision making	371
6.5	Natural sciences	375
6.5.1	Chemistry	375
6.5.2	Biology	376
6.5.3	Physics	378
6.6	Earth sciences	382
6.7	Psychology	385
6.8	Social sciences	390
6.8.1	A historical overview	390
6.8.2	Economics	392
6.8.3	Political science	396
6.9	Computer science	399
6.10	Medicine	407
6.11	Management and business	411
6.12	Other applications	414
7	Significance of Fuzzy Logic	421
7.1	Introduction	421
7.2	A retrospective overview of fuzzy logic	421
7.3	Paradigm shifts in science, mathematics, engineering, and other areas	425
7.4	Assessment of the significance of fuzzy logic	430
7.5	A prospective view on the 50th anniversary of fuzzy logic	442
Appendices		
A	The Enigma of Cox's Proof	449
B	Overview of Classical Logic	453
C	Photographs	469

Glossary of Symbols	473
References	475
Name Index	511
Subject Index	521