Contents

٨؞١	nowle	adamente :			
Acknowledgments ix About the Authors xi					
	hors'	XI			
Aut	liors	Note xiii			
1	Wh	at This Book Is About	1 201		
	1.1	Programming and Mathem	7	Lagrange's Theorem	
	1.2	A Historical Perspective		2 elabora has eshosaT	
	1.3	그 그러워 보고 그는 그 가장 생각이 하지만 때 그리고 살 같아 없었다.	2		
	1.4	Roadmap 4			
2	The	First Algorithm 7			
	2.1	Egyptian Multiplication	8		
	2.2	Improving the Algorithm	11		
	2.3	Thoughts on the Chapter	15		
		Kewara 113			
3	Anc	ient Greek Number Theory	17		
	3.1	Geometric Properties of In	tegers	Seneralizing the Operation	
	3.2	Sifting Primes 20			
	3.3	Implementing and Optimiz	ing the Co	de 23	
	3.4	Perfect Numbers 28			
	3.5	The Pythagorean Program	32		
	3.6	A Fatal Flaw in the Program	n 34		
	3.7	Thoughts on the Chapter	38		
4	Eucl	id's Algorithm 41			
	4.1	Athens and Alexandria	41		
	4.2	Euclid's Greatest Common		Magrithm 15	
	4.3	A Millennium without Math		50	
	4.4	The Strange History of Zero	51		
	4.5	Remainder and Quotient Al	gorithms	53	
4	4.6	Sharing the Code 57			
	4.7	Validating the Algorithm			
	4.8	Thoughts on the Chapter	61	The First Theorem 15	

9.3 Euclid and the Axiomatic Method

5	The E	mergence of Modern Number Theory 63
•	5.1	Mersenne Primes and Fermat Primes 63
		Fermat's Little Theorem 69
		Cancellation 72
	5.4	Proving Fermat's Little Theorem 77
		Euler's Theorem 79
		Applying Modular Arithmetic 83
		Thoughts on the Chapter 84
	3.7	
6	Abst	raction in Mathematics 85
	6.1	Groups 85
	6.2	Monoids and Semigroups 89
	6.3	Some Theorems about Groups 92
	6.4	Subgroups and Cyclic Groups 95
	6.5	Lagrange's Theorem 97
	6.6	Theories and Models 102
	6.7	Examples of Categorical and Non-categorical Theories 104
	6.8	Thoughts on the Chapter 107
7	quesdas	ving a Generic Algorithm 111
7		Villy a delicite Algorithm Doggiroments 111
	7.1	Participants on A 112
	7.2	reduitements and a second seco
	7.3	ar
	7.4	Mew Redaments
	7.5	furning multiply into 1 one.
	7.6	delicializing the operation
	7.7	Computing ribonacor reasses
	7.8	Thoughts on the onapter
8	Moi	re Algebraic Structures 129
	8.1	Stavin Polynomials, and GCD 129
	8.2	Göttingen and German Mathematics 135
	8.3	Noether and the Birth of Abstract Algebra 140
	8.4	AND THE PERSON OF A STATE OF THE PERSON OF T
	8.5	Matrix Multiplication and Semirings 145
	8.6	Chartest Daths 147
	8.7	요한 불명을 병으로 보면 하는 사람들은 사람은 경우를 잃어지는 내용을 살아보고 있다면 하는 것이 없었다. 그리는 사람들은 사람들은 사람들이 되었다면 그렇게 그렇게 그렇게 되었다면 그렇게
	8.8	151
	8.9	ALICENSE DE LA CONTRACTOR DEL CONTRACTOR DE LA CONTRACTOR DE LA CONTRACTOR DE LA CONTRACTOR
-	101-131	vanizing Mathematical Knowledge 155
9		gamzing mathematical throws
	9.1	
	9.2	The First Theorem 159

	9.4 Alternatives to Euclidean Geometry 164
	9.5 Hilbert's Formalist Approach 167
	9.6 Peano and His Axioms 169
	9.7 Building Arithmetic 173
	9.8 Thoughts on the Chapter 176
10	Fundamental Programming Concepts 177
	10.1 Aristotle and Abstraction 177
	10.2 Values and Types 180
	10.3 Concepts 181 845 21500000000000000000000000000000000000
	10.4 Iterators 184
	10.5 Iterator Categories, Operations, and Traits 185
	10.6 Ranges 188
	10.7 Linear Search 190
	10.8 Binary Search 191
	10.9 Thoughts on the Chapter 196
11	Permutation Algorithms 197
	11.1 Permutations and Transpositions 197
	11.2 Swapping Ranges 201 201 200 200 200 200 200 200 200 200
	11.3 Rotation 204
	11.4 Using Cycles 207
	11.5 Reverse 212
	11.6 Space Complexity 215
	11.7 Memory-Adaptive Algorithms 216
	11.8 Thoughts on the Chapter 217
10	Extensions of GCD 219
12	12.1 Hardware Constraints and a More Efficient Algorithm 219
	12.2 Generalizing Stein's Algorithm 222
	12.3 Bézout's Identity 225
	12.4 Extended GCD 229
	12.5 Applications of GCD 234
	12.6 Thoughts on the Chapter 234
	12.0 Thoughts on the chapter 25
13	3 A Real-World Application 237
	13.1 Cryptology 237
	13.2 Primality Testing 240
	13.3 The Miller-Rabin Test 243
	13.4 The RSA Algorithm: How and Why It Works 245
	13.5 Thoughts on the Chapter 248

161

14	Conclusions 249 LAC and particle of the best being a	
	Further Reading 251	
٨	Page and His Axions 159 23 - 24 to some 3 - 4	
Α	Notation 257	
В	Common Proof Techniques 261	
	B.1 Proof by Contradiction 261	
	B.2 Proof by Induction 262	
	B.3 The Pigeonhole Principle 263	
	2 Values and Types 180 S collamedia Not included	
C	C++ for Non-C++ Programmers 265	
	C.1 Template Functions 265	
	C.2 Concepts 266	
	C.3 Declaration Syntax and Typed Constants 267	
	C.4 Function Objects 268	
	C.5 Preconditions, Postconditions, and Assertions 269	
	C.6 STL Algorithms and Data Structures 269	
	C.7 Iterators and Ranges 270	
	C.8 Type Aliases and Type Functions with using in C++11 272	
	C.9 Initializer Lists in C++11 272	
	C.10 Lambda Functions in C++11 272	

Bibliography 275 Index 281

C.11 A Note about inline