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Rendering is a fundamental component of computer graphics. At the highest level of complexity, rendering is the process of converting a description of a three-dimensional scene into an image. Algorithms for animation, geometric modeling, texturing, and other areas of computer graphics all must pass their results through some sort of rendering process so that they can be made visible in an image. Rendering has become a field in its own right, and it has opened new frontiers for computer graphics.		
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In addition to these appendices, there is also a glossary of terms and a bibliography.		
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This book presents a selection of modern rendering algorithms through the documented source code for a complete rendering system. Nearly all of the images in this book, including the one on the front cover, were rendered by this software. All of the algorithms that come together to generate these images are described in these pages. The system, *glrt*, is written using a programming methodology called *literate programming* that mixes prose describing the system with the source code that implements it. We believe that the literate programming approach is a valuable way to introduce ideas in computer graphics and computer science in general. Often, some of the subtleties of an algorithm can be unclear or hidden until it is implemented, so seeing an actual implementation is a good way to acquire a solid understanding of that algorithm's details. Indeed, we believe that deep understanding of a small number of algorithms in this manner provides a stronger base for further study of computer graphics than does superficial understanding of many.