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A VOLUME IN THE IEEE PRESS SERIES ON ELECTROMAGNETIC WAVES

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Key topics covered include:

- Analytical methods for planarly, cylindrically and spherically layered media
- Transient waves, including the Cagniard-de Hoop method
- Variational methods for the scalar wave equation and the electromagnetic wave equation
- Mode-matching techniques for inhomogeneous media
- The Dyadic Green's function and its role in simplifying problem-solving in inhomogeneous media
- Integral equation formulations and inverse problems
- Time domain techniques for inhomogeneous media

This book will be of interest to electromagnetics and remote sensing engineers, physicists, scientists, and geophysicists.

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IEEE Press
445 Hoes Lane
P.O. Box 1331
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ISBN 0-7803-4749-8



9 780780 347496

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