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Multivariable Dynamic Calculus on Time Scales

This book offers the reader an overview of recent developments of multivariable dynamic calculus on time scales, taking readers beyond the traditional calculus texts. Covering topics from parameter-dependent integrals to partial differentiation on time scales, the book's nine pedagogically oriented chapters provide a pathway to this active area of research that will appeal to students and researchers in mathematics and the physical sciences. The authors present a clear and well-organized treatment of the concept behind the mathematics and solution techniques, including many practical examples and exercises.

Mathematics

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