

Contents

<i>List of Figures</i>	<i>page</i> xxii
<i>List of Tables</i>	xxiv
<i>Preface</i>	xxix
1 Introduction and Overview	1
1.1 Introduction	1
1.2 Input–Output Analysis: The Basic Framework	2
1.3 Outline for this Text	3
1.4 Internet Website and Text Locations of Real Datasets	8
References	9
2 Foundations of Input–Output Analysis	10
2.1 Introduction	10
2.2 Notation and Fundamental Relationships	10
2.2.1 Input–Output Transactions and National Accounts	13
2.2.2 Production Functions and the Input–Output Model	15
2.3 An Illustration of Input–Output Calculations	21
2.3.1 Numerical Example: Hypothetical Figures – Approach I	21
<i>Impacts on Industry Outputs</i>	21
<i>Other Impacts</i>	24
2.3.2 Numerical Example: Hypothetical Figures – Approach II	26
2.3.3 Numerical Example: Mathematical Observations	27
2.3.4 Numerical Example: The US 2003 Data	29
2.4 The Power Series Approximation of $(\mathbf{I} - \mathbf{A})^{-1}$	31
2.5 Open Models and Closed Models	34
2.6 The Price Model	41
2.6.1 Overview	41
2.6.2 Physical vs. Monetary Transactions	42
2.6.3 The Price Model based on Monetary Data	43

2.6.4	Numerical Examples Using the Price Model based on Monetary Data	44
	<i>Example 1: Base Year Prices</i>	44
	<i>Example 2: Changed Base Year Prices</i>	45
2.6.5	Applications	46
2.6.6	The Price Model based on Physical Data	47
	<i>Introduction of Prices</i>	48
	<i>Relationship between A and C</i>	49
2.6.7	Numerical Examples Using the Price Model based on Physical Data	50
	<i>Example 1: Base Year Prices</i>	50
	<i>Example 2: Changed Base Year Prices</i>	51
2.6.8	The Quantity Model based on Physical Data	51
2.6.9	A Basic National Income Identity	53
2.7	Summary	53
Appendix 2.1	The Relationship between Approaches I and II	54
A2.1.1	Approach I	54
A2.1.2	Approach II	55
Appendix 2.2	The Hawkins–Simon Conditions	58
	Problems	62
	References	66
3	Input–Output Models at the Regional Level	69
3.1	Introduction	69
3.2	Single-Region Models	70
3.2.1	National Coefficients	70
3.2.2	Regional Coefficients	72
3.2.3	Closing a Regional Model with respect to Households	74
3.3	Many-Region Models: The Interregional Approach	76
3.3.1	Basic Structure of Two-Region Interregional Input–Output Models	77
3.3.2	Interregional Feedbacks in the Two-Region Model	80
3.3.3	Numerical Example: Hypothetical Two-Region Interregional Case	82
3.3.4	Interregional Models with more than Two Regions	86
3.3.5	Implementation of the IRIO Model	87
3.4	Many-Region Models: The Multiregional Approach	87
3.4.1	The Regional Tables	87
3.4.2	The Interregional Tables	89
3.4.3	The Multiregional Model	91
3.4.4	Numerical Example: Hypothetical Two-Region Multiregional Case	93

3.4.5	The US MRIO Models	96
3.4.6	Numerical Example: The Chinese Multiregional Model for 2000	97
3.5	The Balanced Regional Model	101
3.5.1	Structure of the Balanced Regional Model	101
3.5.2	Numerical Example	104
3.6	The Spatial Scale of Regional Models	105
3.7	Summary	106
Appendix 3.1	Basic Relationships in the Multiregional Input–Output Model	107
Appendix 3.2	Sectoral and Regional Aggregation in the 2000 Chinese Multiregional Model	109
Appendix 3.3	The Balanced Regional Model and the Inverse of a Partitioned $(\mathbf{I} - \mathbf{A})$ Matrix	110
	Problems	111
	References	115
4	Organization of Basic Data for Input–Output Models	119
4.1	Introduction	119
4.2	Observations on Ad Hoc Survey-Based Input–Output Tables	119
4.3	Observations on Common Methods for Generating Input–Output Tables	120
4.4	A System of National Economic Accounts	121
4.4.1	The Circular Flow of Income and Consumer Expenditure	122
4.4.2	Savings and Investment	123
4.4.3	Adding Overseas Transactions: Imports, Exports, and Other Transactions	126
4.4.4	The Government Sector	127
4.4.5	The Consolidated Balance Statement for National Accounts	128
4.4.6	Expressing Net Worth	131
4.5	National Income and Product Accounting Conventions	133
4.6	Assembling Input–Output Accounts: The US Case	134
4.7	Additional Considerations	137
4.7.1	Secondary Production: Method of Reallocation	140
	<i>Example 1: Reallocation of Secondary Production</i>	141
4.7.2	Secondary Production: Commodity-by-Industry Accounting	142
	<i>Example 2: Commodity-by-Industry Accounts</i>	142
4.7.3	Reconciling with the National Accounts	143
4.7.4	Producers' and Consumers' Prices	144
	<i>Example 3: Trade and Transportation Margins</i>	146
4.7.5	Accounting for Imports and Exports	149
	<i>Example 4: Competitive and Noncompetitive Imports</i>	149

4.7.6	Removing Competitive Imports from Total Transactions Tables	150
	<i>Approximation Method I</i>	151
	<i>Approximation Method II</i>	151
	<i>Example 5: Import Scrubbing</i>	152
	<i>Implications of the Estimating Assumptions</i>	154
4.7.7	Adjustments for Inventory Change	157
4.7.8	Adjustments for Scrap	157
4.8	Valuation and Double Deflation	157
	<i>Example 6: Double Deflation</i>	159
4.9	The Aggregation Problem: Level of Detail in Input–Output Tables	160
4.9.1	The Aggregation Matrix	161
	<i>Example 7: Sectoral Aggregation</i>	162
4.9.2	Measures of Aggregation Bias	165
4.10	Summary	168
Appendix 4.1	Spatial Aggregation in IRIO and MRIO Models	168
A4.1.1	Spatial Aggregation of IRIO Models	168
A4.1.2	Spatial Aggregation of MRIO Models	172
	Problems	176
	References	180
5	The Commodity-by-Industry Approach in Input–Output Models	184
5.1	Introduction	184
5.1.1	The Use Matrix	185
5.1.2	The Make Matrix	186
5.2	The Basic Accounting Relationships	187
5.3	Technology and Total Requirements Matrices in the Commodity–Industry Approach	188
5.3.1	Industry Source of Commodity Outputs	189
5.3.2	Commodity Composition of Industry Outputs	189
5.3.3	Generating Total Requirements Matrices	189
	<i>Using D</i>	190
	<i>Using C</i>	191
5.3.4	“Industry-Based” Technology	192
5.3.5	“Commodity-Based” Technology	193
5.3.6	Direct Requirements (Technical Coefficients) Matrices	
	Derived from Basic Data	195
5.3.7	Total Requirements Matrices	196
	<i>Approach I: Starting with Technical Coefficients</i>	196
	<i>Approach II: Avoiding $\mathbf{C}^{-1}$ in Commodity Technology Cases</i>	198
	<i>Is Singularity Likely to be a Problem in Real-World Models?</i>	199
5.4	Numerical Examples of Alternative Direct and Total Requirements Matrices	201

5.4.1	Direct Requirements Matrices	202
5.4.2	Total Requirements Matrices	202
	<i>Commodity-Demand Driven Models</i>	202
	<i>Industry-Demand Driven Models</i>	202
5.5	Negative Elements in the Commodity–Industry Framework	203
5.5.1	Commodity Technology	203
	<i>Direct Requirements Matrices</i>	203
	<i>Transactions Matrices</i>	205
	<i>Total Requirements Matrices</i>	206
5.5.2	Industry Technology	207
	<i>Direct Requirements Matrices</i>	207
	<i>Total Requirements Matrices</i>	207
5.5.3	Making a Model Choice	208
	<i>Which Model to Choose?</i>	208
	<i>Dealing with Negative Values</i>	209
5.6	Nonsquare Commodity–Industry Systems	211
5.6.1	Commodity Technology	211
5.6.2	Industry Technology	212
	<i>Direct Requirements Matrices</i>	212
	<i>Total Requirements Matrices</i>	213
5.7	Mixed Technology in the Commodity–Industry Framework	213
5.7.1	Commodity Technology in V_1	216
5.7.2	Industry Technology in V_1	218
5.7.3	Numerical Examples with Mixed Technology	
	Assumptions	219
	<i>Example 1: Commodity Technology in V_1</i>	219
	<i>Example 2: Industry Technology in V_1</i>	220
5.7.4	Additional Mixed Technology Variants	220
5.8	Summary	222
Appendix 5.1	Alternative Approaches to the Derivation of Transactions Matrices	223
A5.1.1	Industry Technology	224
	<i>Commodity-by-Commodity Requirements</i>	224
	<i>Industry-by-Industry Requirements</i>	225
A5.1.2	Commodity Technology	226
	<i>Commodity-by-Commodity Requirements</i>	226
	<i>Industry-by-Industry Requirements</i>	228
Appendix 5.2	Elimination of Negatives in Commodity Technology Models	229
A5.2.1	The Problem	229
	<i>3 × 3 Example</i>	229
	<i>4 × 4 Example</i>	229
	<i>5 × 5 Example (from Almon, 2000)</i>	230

	<i>Final Demand for Goods Made in a Particular Region</i>	267
	<i>More Than Two Regions</i>	268
6.4	Miyazawa Multipliers	271
6.4.1	Disaggregated Household Income Groups	271
6.4.2	Miyazawa's Derivation	273
6.4.3	Numerical Example	275
6.4.4	Adding a Spatial Dimension	276
6.5	Gross and Net Multipliers in Input–Output Models	278
6.5.1	Introduction	278
6.5.2	Multipliers in the Net Input–Output Model	278
	<i>Numerical Example</i>	280
6.5.3	Additional Multiplier Variants	280
	<i>(Indirect Effects)/(Direct Effects)</i>	280
	<i>"Growth Equalized" Multipliers</i>	281
	<i>Another Kind of Net Multiplier</i>	282
6.6	Multipliers and Elasticities	283
6.6.1	Output Elasticity	283
6.6.2	Output-to-Output Multipliers and Elasticities	283
	<i>Direct Effects</i>	283
	<i>Total Effects</i>	284
6.7	Multiplier Decompositions	285
6.7.1	Fundamentals	285
6.7.2	Decompositions in an Interregional Context	286
6.7.3	Stone's Additive Decomposition	288
6.7.4	A Note on Interregional Feedbacks	289
6.7.5	Numerical Illustration	290
6.8	Summary	294
Appendix 6.1	The Equivalence of Total Household Income Multipliers and the Elements in the Bottom Row of $(\mathbf{I} - \bar{\mathbf{A}})^{-1}$	295
Appendix 6.2	Relationship Between Type I and Type II Income Multipliers	296
	Problems	297
	References	299
7	Nonsurvey and Partial-Survey Methods: Fundamentals	303
7.1	Introduction	303
7.2	The Question of Stability of Input–Output Data	303
7.2.1	Stability of National Coefficients	304
	<i>Comparisons of Direct-Input Coefficients</i>	305
	<i>Comparisons of Leontief Inverse Matrices</i>	305
	<i>Other Summary Measures</i>	307
	<i>Data for the US Economy</i>	307

7.2.2	Constant versus Current Prices	307
7.2.3	Stability of Regional Coefficients	309
7.2.4	Summary	310
7.3	Updating and Projecting Coefficients: Trends, Marginal Coefficients and Best Practice Methods	311
7.3.1	Trends and Extrapolation	311
7.3.2	Marginal Input Coefficients	311
7.3.3	“Best Practice” Firms	312
7.4	Updating and Projecting Coefficients: The RAS Approach and Hybrid Methods	313
7.4.1	The RAS Technique	313
7.4.2	Example of the RAS Procedure	320
7.4.3	Updating Coefficients vs. Transactions	327
	<i>Numerical Illustration</i>	327
7.4.4	An Economic Interpretation of the RAS Procedure	328
7.4.5	Incorporating Additional Exogenous Information in an RAS Calculation	330
7.4.6	Modified Example: One Coefficient Known in Advance	331
7.4.7	Hybrid Models: RAS with Additional Information	333
7.4.8	The Constrained Optimization Context	334
7.4.9	Infeasible Problems	335
7.5	Summary	336
Appendix 7.1	RAS as a Solution to the Constrained Minimum Information Distance Problem	337
	Problems	338
	References	343
8	Nonsurvey and Partial-Survey Methods: Extensions	347
8.1	Introduction	347
8.2	Location Quotients and Related Techniques	349
8.2.1	Simple Location Quotients	349
8.2.2	Purchases-Only Location Quotients	353
8.2.3	Cross-Industry Quotients	353
8.2.4	The Semilogarithmic Quotient and its Variants, FLQ and AFLQ	354
8.2.5	Supply–Demand Pool Approaches	356
8.2.6	Fabrication Effects	356
8.2.7	Regional Purchase Coefficients	357
8.2.8	“Community” Input–Output Models	358
8.2.9	Summary	359
8.3	RAS in a Regional Setting	360
8.4	Numerical Illustration	361

8.5	Exchanging Coefficients Matrices	363
8.6	Estimating Interregional Flows	364
8.6.1	Gravity Model Formulations	365
8.6.2	Two-Region Interregional Models	366
8.6.3	Two-Region Logic with more than Two Regions	367
8.6.4	Estimating Commodity Inflows to a Substate Region	369
8.6.5	Additional Studies	371
	<i>Commodity Flows among US States</i>	371
	<i>An Optimization Model for Interregional Flows</i>	372
8.7	Hybrid Methods	373
8.7.1	Generation of Regional Input–Output Tables (GRIT)	374
8.7.2	Double-Entry Bi-Regional Input–Output Tables (DEBRIOT)	375
8.7.3	The Multiregional Input–Output Model for China, 2000 (CMRIO)	377
8.8	International Input–Output Models	378
8.8.1	Introduction	378
8.8.2	Asian International Input–Output Tables	378
8.8.3	“Hybrid” Many-Region Models for the EC	380
8.8.4	China–Japan “Transnational Interregional” Input–Output (TIIO) Model, 2000	381
	<i>Chinese Exports to Japan for Intermediate Demand Applications</i>	381
		383
8.8.5	Leontief’s World Model	383
8.9	The Reconciliation Issue	384
8.10	Summary	386
Appendix 8.1	Geographical Classifications in the World Input–Output Model	387
	Problems	387
	References	392
9	Energy Input–Output Analysis	399
9.1	Introduction	399
9.1.1	Early Approaches to Energy Input–Output Analysis	400
9.1.2	Contemporary Energy Input–Output Analysis	400
9.2	Overview Concepts of Energy Input–Output Analysis	401
9.2.1	The Basic Formulation	403
9.2.2	The Total Energy Requirements Matrix	404
	<i>Example 9.1: Two-Sector Illustration of Hybrid Units Input–Output Analysis</i>	407
	<i>Example 9.2: Generalization to Several Energy Types</i>	408
9.2.3	The Hybrid Units Formulation and Energy Conservation Conditions	410

	<i>Example 9.2: Generalization to Several Energy Types (Revisited)</i>	411
9.3	Further Methodological Considerations	411
9.3.1	Adjusting for Energy Conversion Efficiencies	412
	<i>Example 9.3: Adjusting for Energy Conversion Efficiencies</i>	412
9.3.2	Accounting for Imports	413
9.3.3	Commodity-by-Industry Energy Models	413
9.4	Applications	414
9.4.1	Net Energy Analysis	414
	<i>Example 9.4: Net Energy Analysis</i>	415
9.4.2	Energy Cost of Goods and Services	417
9.4.3	Impacts of New Energy Technologies	421
9.4.4	An Energy Tax	421
9.4.5	Energy and Structural Change	421
9.4.6	Energy Input–Output and Econometrics	423
9.4.7	Other Applications	427
9.5	Summary	427
Appendix 9.1	Earlier Formulation of Energy Input–Output Models	428
A9.1.1	Introduction	428
A9.1.2	Illustration of the Implications of the Traditional Approach	430
	<i>Example 9.5: Energy Input–Output Alternative Formulation</i>	430
	<i>Example 9.6: Energy Input–Output Example (Revised)</i>	431
	<i>Extensions of Example 9.1</i>	433
A9.1.3	General Limitations of the Alternative Formulation	437
Problems		437
References		442
10	Environmental Input–Output Analysis	446
10.1	Introduction	446
10.2	Basic Considerations	446
10.3	Generalized Input–Output Analysis: Basic Framework	447
10.3.1	Accounting for Pollution Impacts	447
10.3.2	Generalized Impacts	447
	<i>Example 10.1: Generalized Input–Output Analysis</i>	448
10.3.3	Summary: Generalized Input–Output Formulations	451
	<i>Case I: Impact Analysis Form</i>	451
	<i>Case II: Planning Form</i>	452
10.4	Generalized Input–Output Analysis: Extensions of the Planning Approach	452
10.4.1	Linear Programming: A Brief Introduction by Means of the Leontief Model	452
10.4.2	Multiple Objectives	457

10.4.3	Conflicting Objectives and Linear Goal Programming	457
10.4.4	Additional Observations	461
	<i>Specifying Objectives</i>	461
	<i>Tightly Constrained Problems</i>	462
	<i>Solution Methods</i>	462
10.4.5	Applications to the Generalized Input–Output Planning Problem	463
10.4.6	Policy Programming	469
	<i>Impact Analysis Form</i>	470
	<i>Planning Form</i>	470
10.4.7	Ecological Commodities	473
10.5	An Augmented Leontief Model	475
10.5.1	Pollution Generation	475
10.5.2	Pollution Elimination	478
	<i>Example 10.2: Pollution-Activity-Augmented Leontief Model</i>	479
10.5.3	Existence of Non-negative Solutions	480
	<i>Example 10.2 (Revisited): Pollution-Activity-Augmented Leontief Model</i>	482
10.6	Economic–Ecologic Models	483
10.6.1	Fully Integrated Models	483
10.6.2	Limited Economic–Ecologic Models	484
	<i>Economic Subsystem</i>	484
	<i>Ecologic Subsystem</i>	485
	<i>Commodity-by-Industry Formulation</i>	485
	<i>Example 10.3: Limited Economic–Ecologic Models</i>	485
10.7	Pollution Dispersion	487
10.7.1	Gaussian Dispersion Models	487
10.7.2	Coupling Pollution Dispersion and Input–Output Models	488
	<i>Example 10.4: Coupling Input–Output and Pollution Dispersion Models</i>	488
10.8	Other Applications	489
10.9	Summary	490
	Problems	490
	References	494
11	Social Accounting Matrices	499
11.1	Introduction	499
11.2	Social Accounting Matrices: Background	499
11.3	Social Accounting Matrices: Basic Concepts	501
11.4	The Households Account	502
11.5	The Value-Added Account	504

11.6	Interindustry Transactions and the Connection to the Input–Output Framework	504
11.7	Expanding the Social Accounts	507
11.8	Additional Social Accounting Variables	507
11.9	A “Fully Articulated” SAM	510
11.10	SAM Multipliers	513
11.10.1	SAM Multipliers: Basic Structure	514
11.10.2	Decomposition of SAM Multipliers	516
	<i>Example 11.1: Reduced Form Case</i>	518
11.10.3	Multipliers in an Expanded SAM	522
	<i>Example 11.2: The Expanded Case</i>	524
11.10.4	Additive Multipliers	528
11.11	The Relationship between Input–Output and SAM Multipliers	530
11.12	Balancing SAM Accounts	535
11.12.1	Example: Balancing a SAM	535
11.12.2	Example: Balancing a SAM with Additional Information	536
11.13	Some Applications of SAMs	536
11.14	Summary	537
	Problems	537
	References	541
12	Supply-Side Models, Linkages, and Important Coefficients	543
12.1	Supply Side Input–Output Models	543
12.1.1	The Early Interpretation	543
	<i>Numerical Illustration (Hypothetical Data)</i>	546
	<i>Numerical Application (US Data)</i>	547
12.1.2	Relationships between A and B and between L and G	547
12.1.3	Comments on the Early Interpretation	548
12.1.4	Joint Stability	549
	<i>The Issue</i>	549
	<i>Conditions under which both A and B will be Stable</i>	551
12.1.5	Reinterpretation as a Price Model	551
	<i>Connection to the Leontief Price Model (Algebra)</i>	553
	<i>Connection to the Leontief Price Model (Numerical Illustration)</i>	553
	<i>A Ghosh Quantity Model</i>	554
12.2	Linkages in Input–Output Models	555
12.2.1	Backward Linkage	556
12.2.2	Forward Linkage	558
12.2.3	“Net” Backward Linkage	558
12.2.4	Classifying Backward and Forward Linkage Results	559

12.2.5	Spatial Linkages	560
12.2.6	Hypothetical Extraction	563
	<i>Backward Linkage</i>	564
	<i>Forward Linkage</i>	564
12.2.7	Illustration Using US Data	565
12.3	Identifying Important Coefficients	567
12.3.1	Mathematical Background	568
12.3.2	Relative Sizes of Elements in the Leontief Inverse	569
	<i>Observation 1</i>	569
	<i>Observation 2</i>	570
	<i>Observation 3</i>	570
12.3.3	“Inverse-Important” Coefficients	570
12.3.4	Numerical Example	572
12.3.5	Impacts on Gross Outputs	573
12.3.6	Fields of Influence	578
12.3.7	Additional Measures of Coefficient Importance	580
	<i>Converting Output to Employment, Income, etc.</i>	580
	<i>Elasticity Coefficient Analysis</i>	581
	<i>Relative Changes in All Gross Outputs</i>	581
	<i>Impacts of Changes in more than One Element of the A Matrix</i>	582
12.4	Summary	582
Appendix 12.1	The Sherman–Morrison–Woodbury Formulation	582
	A12.1.1 Introduction	582
	A12.1.2 Application to Leontief Inverses	585
	Problems	585
	References	587
13	Structural Decomposition, Mixed and Dynamic Models	593
13.1	Structural Decomposition Analysis	593
13.1.1	Initial Decompositions: Changes in Gross Outputs	593
	<i>Numerical Example</i>	596
13.1.2	Next-Level Decompositions: Digging Deeper into $\Delta \mathbf{f}$ and $\Delta \mathbf{L}$	598
	<i>Additive Decompositions with Products of more than Two Terms</i>	598
	<i>Changes in Final Demand</i>	599
13.1.3	Numerical Examples	601
	<i>One Category of Final Demand ($p = 1$)</i>	601
	<i>Two Categories of Final Demand ($p = 2$)</i>	601
13.1.4	Changes in the Direct Inputs Matrix	602
	<i>Decomposition of $\Delta \mathbf{L}$</i>	602
	<i>Decomposition of $\Delta \mathbf{A}$</i>	604
	<i>Numerical Illustration (continued)</i>	605

13.1.5	Decompositions of Changes in Some Function of $\mathbf{x}$	606
13.1.6	Summary for $\Delta \mathbf{x}$	607
13.1.7	SDA in a Multiregional Input–Output (MRIO) Model	607
13.1.8	Empirical Examples	608
	<i>Studies Using National Models</i>	608
	<i>Studies Using a Single-Region or Connected-Region Model</i>	615
13.2	Mixed Models	621
13.2.1	Exogenous Specification of One Sector's Output	621
	<i>Rearranging the Basic Equations</i>	621
	<i>"Extracting" the Sector</i>	624
13.2.2	An Alternative Approach When $f_1, \dots, f_{n-1}$ and x_n Are Exogenously Specified	625
13.2.3	Examples with x_n Exogenous	626
	<i>Example 1: $f_1 = 100,000, f_2 = 200,000, x_3 = 150,000$</i>	626
	<i>Example 2: $f_1 = f_2 = 0, x_3 = 150,000$</i>	627
	<i>Example 3: $f_1 = 100,000, f_2 = 200,000, x_3 = 100,000$</i>	628
	<i>Example 4: The Critical Value of x_3</i>	628
	<i>Multipliers</i>	629
13.2.4	Exogenous Specification of $f_1, \dots, f_k, x_{k+1}, \dots, x_n$	629
13.2.5	An Example with x_{n-1} and x_n Exogenous	632
	<i>Example 5 (Example 2 expanded)</i>	632
13.3	New Industry Impacts in the Input–Output Model	633
13.3.1	New Industry: The Final-Demand Approach	634
13.3.2	New Industry: Complete Inclusion in the Technical Coefficients Matrix	636
13.3.3	A New Firm in an Existing Industry	637
13.3.4	Other Structural Changes	639
13.4	Dynamic Considerations in Input–Output Models	639
13.4.1	General Relationships	639
13.4.2	A Three-Period Example	642
	<i>Terminal Conditions</i>	643
	<i>Initial Conditions</i>	644
13.4.3	Numerical Example 1	645
	<i>Terminal Conditions</i>	646
	<i>Initial Conditions</i>	648
13.4.4	Numerical Example 2	649
	<i>Terminal Conditions</i>	649
	<i>Initial Conditions</i>	649
13.4.5	"Dynamic" Multipliers	650
13.4.6	Turnpike Growth and Dynamic Models	651
	<i>Example</i>	653

13.4.7	Alternative Input–Output Dynamics	653
13.5	Summary	654
Appendix 13.1	Alternative Decompositions of $\mathbf{x} = \mathbf{LBf}$	655
Appendix 13.2	Exogenous Specification of Some Elements of $\mathbf{x}$	656
A13.2.1	The General Case: An n -sector Model with k Endogenous Outputs	656
A13.2.2	The Output-to-Output Multiplier Matrix	658
A13.2.3	The Inverse of a Partitioned $(\mathbf{I} - \mathbf{A}^{(n)})$ Matrix	659
A13.2.4	The Case of $k = 2, n = 3$	659
A13.2.5	The Case of $k = 1, n = 3$	660
A13.2.6	“Extracting” the Last $(n - k)$ Sectors	662
	Problems	663
	References	665
14	Additional Topics	669
14.1	Introduction	669
14.2	Input–Output and Measuring Economic Productivity	670
14.2.1	Total Factor Productivity	670
14.2.2	Numerical Example: Total Factor Productivity	672
14.2.3	Accounting for Prices	673
14.2.4	References for Section 14.2	674
14.3	Graph Theory, Structural Path Analysis, and Qualitative Input–Output Analysis (QIOA)	674
14.3.1	References for Section 14.3	677
14.4	Fundamental Economic Structure (FES)	678
14.4.1	References for Section 14.4	679
14.5	Input–Output, Econometrics, and Computable General Equilibrium Models	679
14.5.1	The Variable Input–Output Model	680
14.5.2	Regional Input–Output Econometric Models	681
14.5.3	Computable General Equilibrium Models	681
14.5.4	References for Section 14.5	682
14.6	Additional Resources for Input–Output Extensions and Applications	683
14.6.1	Edited Collections	684
14.6.2	Journal Special Issues	685
14.6.3	Collections of Reprints	686
14.6.4	References for Section 14.6	686
14.7	Some Concluding Reflections	686

Appendix A Matrix Algebra for Input–Output Models	688
A.1 Introduction	688
A.2 Matrix Operations: Addition and Subtraction	689
A.2.1 Addition	689
A.2.2 Subtraction	689
A.2.3 Equality	689
A.2.4 The Null Matrix	689
A.3 Matrix Operations: Multiplication	689
A.3.1 Multiplication of a Matrix by a Number	689
A.3.2 Multiplication of a Matrix by Another Matrix	689
A.3.3 The Identity Matrix	690
A.4 Matrix Operations: Transposition	691
A.5 Representation of Linear Equation Systems	691
A.6 Matrix Operations: Division	693
A.7 Diagonal Matrices	696
A.8 Summation Vectors	698
A.9 Matrix Inequalities	698
A.10 Partitioned Matrices	699
A.10.1 Multiplying Partitioned Matrices	699
A.10.2 The Inverse of a Partitioned Matrix	700
References	701
 Appendix B Reference Input–Output Tables for the United States (1919–2006)	702
B.1 Introduction	702
B.2 Transactions Accounts	703
B.3 Matrices of Technical Coefficients and Total Requirements	715
References for US Input–Output Tables (1919–2006)	722
 Appendix C Historical Notes on the Development of Leontief’s Input–Output Analysis	724
C.1 Conceptual Foundations	724
C.2 Quesnay and the Physiocrats	725
C.3 Mathematical Formalization	728
C.4 Leontief and the “Economy as a Circular Flow”	729
C.5 Development of Input–Output Analysis	731
References	735

<i>Author Index</i>	738
<i>Subject Index</i>	746

**The Following Supplementary Appendices are Available Online at
www.cambridge.org/millerandblair**

**Web Appendix 5W.1 Left and Right Inverses in Nonsquare Input–Output
Systems**

- A5W.1.1 Introduction
- A5W.1.2 More Commodities than Industries ($m > n$)
 - Numerical Illustration*
 - Commodity Technology*
 - Industry Technology*
- A5W.1.3 Fewer Commodities than Industries ($m < n$)
 - Numerical Illustration*
 - Commodity Technology*
 - Industry Technology*

**Web Appendix 8W.1 Detailed Results for the Numerical Illustration in
Section 8.4**

Web Appendix 12W.1 Hypothetical Extractions with Partitioned Matrices

- A12W.1.1 Case 1: Complete Extraction of Sector 1
- A12W.1.2 Case 2: Extraction of Sector 1's Intersectoral Relations
- A12W.1.3 Case 3: Extraction of Sector 1's Intermediate Purchases
- A12W.1.4 Case 4: Extraction of Sector 1's Intermediate Sales
- A12W.1.5 Case 5: Extraction of Sector 1's Intersectoral Intermediate
Purchases
- A12W.1.6 Case 6: Extraction of Sector 1's Intersectoral Intermediate
Sales
- A12W.1.7 The Ghosh Model and Some Comparisons

**Web Appendix 12W.2 Brief History of Leontief Inverses with Errors in the
Coefficients of A**

- A12W.2.1 Mathematical Background
- A12W.2.2 Application of Leontief Inverses
 - Dwyer and Waugh (1953)*
 - Evans (1954)*
 - West (1982)*
 - Sherman-Morrison and Sonis-Hewings*