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Since energy, by its very nature a directed phenomenon, it seems probable that two scalar quantities should be sufficient to determine the motion. The energy theorem, which states that the sum of the kinetic and potential energies remains unchanged

This sum of the two does not necessarily imply that vectorial quantities are used.

¹See note on terminology at end of chap. 1, section 1.