

<i>Contents of Basic Algebra</i>	x
<i>Preface</i>	xi
<i>List of Figures</i>	xv
<i>Dependence among Chapters</i>	xvi
<i>Guide for the Reader</i>	xvii
<i>Notation and Terminology</i>	xxi

I. TRANSITION TO MODERN NUMBER THEORY	1
1. Historical Background	1
2. Quadratic Reciprocity	8
3. Equivalence and Reduction of Quadratic Forms	12
4. Composition of Forms, Class Group	24
5. Genera	31
6. Quadratic Number Fields and Their Units	35
7. Relationship of Quadratic Forms to Ideals	38
8. Primes in the Progressions $4n + 1$ and $4n + 3$	50
9. Dirichlet Series and Euler Products	56
10. Dirichlet's Theorem on Primes in Arithmetic Progressions	61
11. Problems	67
II. WEDDERBURN-ARTIN RING THEORY	76
1. Historical Motivation	77
2. Semisimple Rings and Wedderburn's Theorem	81
3. Rings with Chain Condition and Artin's Theorem	87
4. Wedderburn-Artin Radical	89
5. Wedderburn's Main Theorem	94
6. Semisimplicity and Tensor Products	104
7. Skolem-Noether Theorem	111
8. Double Centralizer Theorem	114
9. Wedderburn's Theorem about Finite Division Rings	117
10. Frobenius's Theorem about Division Algebras over the Reals	118
11. Problems	120

III. BRAUER GROUP	123
1. Definition and Examples, Relative Brauer Group	124
2. Factor Sets	132
3. Crossed Products	135
4. Hilbert's Theorem 90	145
5. Digression on Cohomology of Groups	147
6. Relative Brauer Group when the Galois Group Is Cyclic	158
7. Problems	162
IV. HOMOLOGICAL ALGEBRA	166
1. Overview	167
2. Complexes and Additive Functors	171
3. Long Exact Sequences	184
4. Projectives and Injectives	192
5. Derived Functors	202
6. Long Exact Sequences of Derived Functors	210
7. Ext and Tor	223
8. Abelian Categories	232
9. Problems	250
V. THREE THEOREMS IN ALGEBRAIC NUMBER THEORY	262
1. Setting	262
2. Discriminant	266
3. Dedekind Discriminant Theorem	274
4. Cubic Number Fields as Examples	279
5. Dirichlet Unit Theorem	288
6. Finiteness of the Class Number	298
7. Problems	307
VI. REINTERPRETATION WITH ADELES AND IDELES	313
1. p -adic Numbers	314
2. Discrete Valuations	320
3. Absolute Values	331
4. Completions	342
5. Hensel's Lemma	349
6. Ramification Indices and Residue Class Degrees	353
7. Special Features of Galois Extensions	368
8. Different and Discriminant	371
9. Global and Local Fields	382
10. Adeles and Ideles	388
11. Problems	397

VII. INFINITE FIELD EXTENSIONS	403
1. Nullstellensatz	404
2. Transcendence Degree	408
3. Separable and Purely Inseparable Extensions	414
4. Krull Dimension	423
5. Nonsingular and Singular Points	428
6. Infinite Galois Groups	434
7. Problems	445
VIII. BACKGROUND FOR ALGEBRAIC GEOMETRY	447
1. Historical Origins and Overview	448
2. Resultant and Bezout's Theorem	451
3. Projective Plane Curves	456
4. Intersection Multiplicity for a Line with a Curve	466
5. Intersection Multiplicity for Two Curves	473
6. General Form of Bezout's Theorem for Plane Curves	488
7. Gröbner Bases	491
8. Constructive Existence	499
9. Uniqueness of Reduced Gröbner Bases	508
10. Simultaneous Systems of Polynomial Equations	510
11. Problems	516
IX. THE NUMBER THEORY OF ALGEBRAIC CURVES	520
1. Historical Origins and Overview	520
2. Divisors	531
3. Genus	534
4. Riemann–Roch Theorem	540
5. Applications of the Riemann–Roch Theorem	552
6. Problems	554
X. METHODS OF ALGEBRAIC GEOMETRY	558
1. Affine Algebraic Sets and Affine Varieties	559
2. Geometric Dimension	563
3. Projective Algebraic Sets and Projective Varieties	570
4. Rational Functions and Regular Functions	579
5. Morphisms	590
6. Rational Maps	595
7. Zariski's Theorem about Nonsingular Points	600
8. Classification Questions about Irreducible Curves	604
9. Affine Algebraic Sets for Monomial Ideals	618
10. Hilbert Polynomial in the Affine Case	626

X. METHODS OF ALGEBRAIC GEOMETRY (Continued)

11. Hilbert Polynomial in the Projective Case	633
12. Intersections in Projective Space	635
13. Schemes	638
14. Problems	644
<i>Hints for Solutions of Problems</i>	649
<i>Selected References</i>	713
<i>Index of Notation</i>	717
<i>Index</i>	721

CONTENTS OF *BASIC ALGEBRA*

I. Preliminaries about the Integers, Polynomials, and Matrices	
II. Vector Spaces over \mathbb{Q} , \mathbb{R} , and \mathbb{C}	
III. Inner-Product Spaces	
IV. Groups and Group Actions	
V. Theory of a Single Linear Transformation	
VI. Multilinear Algebra	
VII. Advanced Group Theory	
VIII. Commutative Rings and Their Modules	
IX. Fields and Galois Theory	
X. Modules over Noncommutative Rings	