

CONTENTS

<i>Contents of Advanced Algebra</i>	x
<i>List of Figures</i>	xi
<i>Preface to the Revised Edition</i>	xiii
<i>Preface to the Original Edition</i>	xv
<i>Dependence Among Chapters</i>	xix
<i>Standard Notation</i>	xx
<i>Guide for the Reader</i>	xxi
I. PRELIMINARIES ABOUT THE INTEGERS, POLYNOMIALS, AND MATRICES	1
1. Division and Euclidean Algorithms	1
2. Unique Factorization of Integers	4
3. Unique Factorization of Polynomials	9
4. Permutations and Their Signs	15
5. Row Reduction	19
6. Matrix Operations	24
7. Problems	30
II. VECTOR SPACES OVER $\mathbb{Q}$, $\mathbb{R}$, AND $\mathbb{C}$	33
1. Spanning, Linear Independence, and Bases	33
2. Vector Spaces Defined by Matrices	38
3. Linear Maps	42
4. Dual Spaces	50
5. Quotients of Vector Spaces	54
6. Direct Sums and Direct Products of Vector Spaces	58
7. Determinants	65
8. Eigenvectors and Characteristic Polynomials	73
9. Bases in the Infinite-Dimensional Case	78
10. Problems	82
III. INNER-PRODUCT SPACES	89
1. Inner Products and Orthonormal Sets	89
2. Adjoint	99
3. Spectral Theorem	105
4. Problems	112

IV. GROUPS AND GROUP ACTIONS	117
1. Groups and Subgroups	118
2. Quotient Spaces and Homomorphisms	129
3. Direct Products and Direct Sums	135
4. Rings and Fields	141
5. Polynomials and Vector Spaces	148
6. Group Actions and Examples	159
7. Semidirect Products	167
8. Simple Groups and Composition Series	171
9. Structure of Finitely Generated Abelian Groups	176
10. Sylow Theorems	185
11. Categories and Functors	189
12. Problems	200
V. THEORY OF A SINGLE LINEAR TRANSFORMATION	211
1. Introduction	211
2. Determinants over Commutative Rings with Identity	215
3. Characteristic and Minimal Polynomials	218
4. Projection Operators	226
5. Primary Decomposition	228
6. Jordan Canonical Form	231
7. Computations with Jordan Form	238
8. Problems	241
VI. MULTILINEAR ALGEBRA	248
1. Bilinear Forms and Matrices	249
2. Symmetric Bilinear Forms	253
3. Alternating Bilinear Forms	256
4. Hermitian Forms	258
5. Groups Leaving a Bilinear Form Invariant	260
6. Tensor Product of Two Vector Spaces	263
7. Tensor Algebra	277
8. Symmetric Algebra	283
9. Exterior Algebra	291
10. Problems	295
VII. ADVANCED GROUP THEORY	306
1. Free Groups	306
2. Subgroups of Free Groups	317
3. Free Products	322
4. Group Representations	329

VII. ADVANCED GROUP THEORY (Continued)

5. Burnside's Theorem	345
6. Extensions of Groups	347
7. Problems	360

VIII. COMMUTATIVE RINGS AND THEIR MODULES

	370
1. Examples of Rings and Modules	370
2. Integral Domains and Fields of Fractions	381
3. Prime and Maximal Ideals	384
4. Unique Factorization	387
5. Gauss's Lemma	393
6. Finitely Generated Modules	399
7. Orientation for Algebraic Number Theory and Algebraic Geometry	411
8. Noetherian Rings and the Hilbert Basis Theorem	417
9. Integral Closure	420
10. Localization and Local Rings	428
11. Dedekind Domains	437
12. Problems	443

IX. FIELDS AND GALOIS THEORY

	452
1. Algebraic Elements	453
2. Construction of Field Extensions	457
3. Finite Fields	461
4. Algebraic Closure	464
5. Geometric Constructions by Straightedge and Compass	468
6. Separable Extensions	474
7. Normal Extensions	481
8. Fundamental Theorem of Galois Theory	484
9. Application to Constructibility of Regular Polygons	489
10. Application to Proving the Fundamental Theorem of Algebra	492
11. Application to Unsolvability of Polynomial Equations with Nonsolvable Galois Group	493
12. Construction of Regular Polygons	499
13. Solution of Certain Polynomial Equations with Solvable Galois Group	506
14. Proof That π Is Transcendental	515
15. Norm and Trace	519
16. Splitting of Prime Ideals in Extensions	526
17. Two Tools for Computing Galois Groups	532
18. Problems	539

X. MODULES OVER NONCOMMUTATIVE RINGS	553
1. Simple and Semisimple Modules	553
2. Composition Series	560
3. Chain Conditions	565
4. Hom and End for Modules	567
5. Tensor Product for Modules	574
6. Exact Sequences	583
7. Problems	588
APPENDIX	593
A1. Sets and Functions	593
A2. Equivalence Relations	599
A3. Real Numbers	601
A4. Complex Numbers	604
A5. Partial Orderings and Zorn's Lemma	605
A6. Cardinality	610
<i>Hints for Solutions of Problems</i>	615
<i>Selected References</i>	715
<i>Index of Notation</i>	717
<i>Index</i>	721

CONTENTS OF *ADVANCED ALGEBRA*

I. Transition to Modern Number Theory	
II. Wedderburn–Artin Ring Theory	
III. Brauer Group	
IV. Homological Algebra	
V. Three Theorems in Algebraic Number Theory	
VI. Reinterpretation with Adeles and Ideles	
VII. Infinite Field Extensions	
VIII. Background for Algebraic Geometry	
IX. The Number Theory of Algebraic Curves	
X. Methods of Algebraic Geometry	