

The aim of this book is to present a substantial part of matrix analysis that is functional analytic in spirit. Much of this will be of interest to graduate students and research workers in operator theory, operator algebras, mathematical physics, and numerical analysis.

The book can be used as a basic text for graduate courses on advanced linear algebra and matrix analysis. It can also be used as supplementary text for courses in operator theory and numerical analysis. Among topics covered are the theory of majorization, variational principles for eigenvalues, operator monotone and convex functions, perturbation of matrix functions and matrix inequalities. Much of this is presented for the first time in a unified way in a textbook.

The reader will learn several powerful methods and techniques of wide applicability, and see connections with other areas of mathematics. A large selection of matrix inequalities will make this book a valuable reference for students and researchers who are working in numerical analysis, mathematical physics and operator theory.

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