

CONTENTS	v
PREFACE	vii
CHAPTER ONE / PRELIMINARIES	1
1.1 Introduction	1
1.2 A survey of Boolean propositional logic	6
1.3 Boolean predicate calculus	10
1.4 Function symbols; varieties of algebras	15
1.5 Lattices and Boolean algebras	20
1.6 Ordered Abelian groups	22
CHAPTER TWO / MANY-VALUED PROPOSITIONAL CALCULI	27
2.1 Continuous t-norms and their residua	27
2.2 The basic many-valued logic	35
2.3 Residuated lattices; a completeness theorem	46
2.4 Some additional topics	56
CHAPTER THREE / ŁUKASIEWICZ PROPOSITIONAL LOGIC	63
3.1 Getting Łukasiewicz logic	63
3.2 MV-algebras; a completeness theorem	70
3.3 Rational Pavelka logic	79
CHAPTER FOUR / PRODUCT LOGIC, GÖDEL LOGIC	89
4.1 Product logic	89
4.2 Gödel logic	97
4.3 Appendix: Boolean logic	103
CHAPTER FIVE / MANY-VALUED PREDICATE LOGICS	109
5.1 The basic many-valued predicate logic	109
5.2 Completeness	119
5.3 Axiomatizing Gödel logic	124
5.4 Łukasiewicz and product predicate logic	127
5.5 Many-sorted fuzzy predicate calculi	139
5.6 Similarity and equality	141

CHAPTER SIX / COMPLEXITY AND UNDECIDABILITY	149
6.1 Preliminaries	149
6.2 Complexity of fuzzy propositional calculi	154
6.3 Undecidability of fuzzy logics	161
CHAPTER SEVEN / ON APPROXIMATE INFERENCE	167
7.1 The compositional rule of inference	168
7.2 Fuzzy functions and fuzzy controllers	177
7.3 An alternative approach to fuzzy rules	189
CHAPTER EIGHT / GENERALIZED QUANTIFIERS AND MODALITIES	195
8.1 Generalized quantifiers in Boolean logic	195
8.2 Two-valued modal logics	205
8.3 Fuzzy quantifiers and modalities	215
8.4 On “probably” and “many”	228
8.5 More on “probably” and “many”	238
CHAPTER NINE / MISCELLANEA	249
9.1 Takeuti-Titani fuzzy logic	249
9.2 An abstract fuzzy logic	261
9.3 On the liar paradox	265
9.4 Concluding remarks	271
CHAPTER TEN / HISTORICAL REMARKS	277
10.1 Until the forties	277
10.2 The fifties	278
10.3 The sixties	279
10.4 The seventies	279
10.5 The eighties	280
10.6 The nineties	281
REFERENCES	283
INDEX	295