

# Contents

4	Lebesgue integrals . . . . .	367
5	Lebesgue integrals as limits . . . . .	371
6	Italian Measure Theory . . . . .	376
1057	Vitali coverings and density points . . . . .	383
1058	Lebesgue's Fundamental Theorem . . . . .	396
1159	Lebesgue's Last Theorem . . . . .	401
115	Appendix A: Translations and Dilations . . . . .	407
115	Appendix B: The Banach-Tarski Paradox . . . . .	409
855	Appendix C: Riemann integrals and graphs . . . . .	409
332	Appendix D: Littlewood's Three Principles . . . . .	411
045	Appendix E: Roubiness . . . . .	412
845	Appendix F: Continuity . . . . .	413
125	Suggested Reading . . . . .	414
125	Bibliography . . . . .	415

## CONTENTS

<b>1</b>	<b>Real Numbers</b>	<b>1</b>
1	Preliminaries . . . . .	1
2	Cuts . . . . .	10
3	Euclidean Space . . . . .	21
4	Cardinality . . . . .	28
5*	Comparing Cardinalities . . . . .	34
6*	The Skeleton of Calculus . . . . .	36
	Exercises . . . . .	40
<b>2</b>	<b>A Taste of Topology</b>	<b>51</b>
1	Metric Space Concepts . . . . .	51
2	Compactness . . . . .	76
3	Connectedness . . . . .	82
4	Coverings . . . . .	88
5	Cantor Sets . . . . .	95
6*	Cantor Set Lore . . . . .	99
7*	Completion . . . . .	108
	Exercises . . . . .	115

<b>3 Functions of a Real Variable</b>	<b>139</b>
1 Differentiation . . . . .	139
2 Riemann Integration . . . . .	154
3 Series . . . . .	179
Exercises . . . . .	186
<b>4 Function Spaces</b>	<b>201</b>
1 Uniform Convergence and $C^0[a, b]$ . . . . .	201
2 Power Series . . . . .	211
3 Compactness and Equicontinuity in $C^0$ . . . . .	213
4 Uniform Approximation in $C^0$ . . . . .	217
5 Contractions and ODE's . . . . .	228
6* Analytic Functions . . . . .	235
7* Nowhere Differentiable Continuous Functions . . . . .	240
8* Spaces of Unbounded Functions . . . . .	248
Exercises . . . . .	251
<b>5 Multivariable Calculus</b>	<b>267</b>
1 Linear Algebra . . . . .	267
2 Derivatives . . . . .	271
3 Higher derivatives . . . . .	279
4 Smoothness Classes . . . . .	284
5 Implicit and Inverse Functions . . . . .	286
6* The Rank Theorem . . . . .	290
7* Lagrange Multipliers . . . . .	296
8 Multiple Integrals . . . . .	300
9 Differential Forms . . . . .	313
10 The General Stokes' Formula . . . . .	325
11* The Brouwer Fixed Point Theorem . . . . .	334
Appendix A: Perorations of Dieudonné . . . . .	337
Appendix B: The History of Cavalieri's Principle . . . . .	338
Appendix C: A Short Excursion into the Complex Field . . . . .	339
Appendix D: Polar Form . . . . .	340
Appendix E: Determinants . . . . .	342
Exercises . . . . .	345

<b>6</b>	<b>Lebesgue Theory</b>	<b>363</b>
1	Outer measure . . . . .	363
2	Measurability . . . . .	367
3	Regularity . . . . .	371
4	Lebesgue integrals . . . . .	376
5	Lebesgue integrals as limits . . . . .	383
6	Italian Measure Theory . . . . .	387
7	Vitali coverings and density points . . . . .	391
8	Lebesgue's Fundamental Theorem of Calculus . . . . .	396
9	Lebesgue's Last Theorem . . . . .	401
	Appendix A: Translations and Nonmeasurable sets . . . . .	407
	Appendix B: The Banach-Tarski Paradox . . . . .	409
	Appendix C: Riemann integrals as undergraphs . . . . .	409
	Appendix D: Littlewood's Three Principles . . . . .	411
	Appendix E: Roundness . . . . .	412
	Appendix F: Money . . . . .	413
	Suggested Reading . . . . .	414
	Bibliography . . . . .	415
	Exercises . . . . .	417