

# CONTENTS

[Asterisks indicate sections that involve algebraic formulae.]

	page
<b>Chapter I: Hints for Solving Puzzles</b>	
<i>I. Various Kinds of Puzzles</i>	
1. Literary puzzles . . . . .	1
2. Pure puzzles . . . . .	2
3. Remarks on pure puzzles . . . . .	3
4. Puzzle games . . . . .	4
5. Correspondences and differences between puzzles and games . . . . .	4
<i>II. Solving by Trial</i>	
6. Trial and error . . . . .	6
7. Systematic trial . . . . .	7
8. Division into cases . . . . .	7
9. Example of a puzzle tree . . . . .	8
<i>III. Classification System</i>	
10. Choosing a classification system . . . . .	10
11. Usefulness of a classification system . . . . .	10
12. More about the classification system . . . . .	12
<i>IV. Solving a Puzzle by Simplification</i>	
13. Simplifying a puzzle . . . . .	13
14. Example of how to simplify a puzzle . . . . .	13
15. Remarks on the seven coins puzzle . . . . .	15
16. Reversing a puzzle . . . . .	17
17. Example of reversing a puzzle . . . . .	17
<i>V. Solving a Puzzle by Breaking It Up</i>	
18. Breaking a puzzle up into smaller puzzles . . . . .	19
19. Application to the crossing puzzle . . . . .	20
20. Number of solutions of the crossing puzzle . . . . .	22
21. Restrictive condition in the crossing puzzle . . . . .	23
22. Shunting puzzle . . . . .	24
<i>VI. Some Puzzles with Multiples</i>	
*23. Trebles puzzle . . . . .	25
*24. Breaking up the trebles puzzle . . . . .	26
*25. Trebles puzzle with larger numbers . . . . .	30
*26. Doubles puzzle with 7-digit numbers . . . . .	31
*27. Remarks on the numbers of §26 . . . . .	34
*28. Quintuples puzzle . . . . .	35
<b>Chapter II: Some Domino Puzzles</b>	
<i>I. Symmetric Domino Puzzle, with Extensions</i>	
29. Symmetric domino puzzle . . . . .	38
30. Extended symmetric domino puzzle . . . . .	41
*31. Another extension of the symmetric domino puzzle . . . . .	43

	page
<i>II. Doubly Symmetric Domino Puzzle</i>	
*32. First doubly symmetric domino puzzle . . . . .	46
*33. Doubly symmetric domino puzzle without restrictive condition . . . . .	52
*34. Connection with the puzzle of §32 . . . . .	56
35. Second doubly symmetric domino puzzle . . . . .	59
36. Puzzle with dominos in a rectangle . . . . .	60
<i>III. Smallest and Largest Number of Corners</i>	
37. Salient and re-entrant angles . . . . .	64
38. Puzzle with the smallest number of angles . . . . .	64
39. Puzzle with the largest number of angles . . . . .	66
<b>Chapter III: The Game of Noughts and Crosses</b>	
<i>I. Description of the Game</i>	
40. Rules of the game . . . . .	69
41. Supplement to the game . . . . .	69
42. Consequences of the rules . . . . .	70
<i>II. Considerations Affecting Values of the Squares</i>	
43. Value of a square . . . . .	71
44. Remarks on the value of a square . . . . .	72
<i>III. Directions for Good Play</i>	
45. Semi-row or threat . . . . .	72
46. Double threat . . . . .	73
47. Combined threat . . . . .	73
48. Replying to a double threat . . . . .	74
49. Further directions for good play . . . . .	76
<i>IV. Some Remarks on Good Play</i>	
50. Remarks on the double threat . . . . .	78
51. Connection with the value of a move . . . . .	79
<i>V. General Remarks on the Analysis of the Game</i>	
52. Preliminary remarks . . . . .	79
53. Diagrams . . . . .	80
54. Tree derived from the diagrams . . . . .	80
<i>VI. Partial Analysis of the Game</i>	
55. John starts with the central square 5, Peter replies with the corner square 1 . . . . .	81
56. John starts with the corner square 1, Peter replies with the central square 5 . . . . .	82
57. John starts with the border square 2, Peter replies with the central square 5 . . . . .	82
58. Equitable nature of the game . . . . .	83
<i>VII. Complete Analysis of the Game</i>	
59. John starts with the central square 5, Peter replies with the border square 2 . . . . .	84
60. John starts with the corner square 1, Peter replies with the border square 2 . . . . .	84
61. John starts with the corner square 1, Peter replies with the corner square 3 . . . . .	86
62. John starts with the corner square 1, Peter replies with the border square 6 . . . . .	87

page

63. John starts with the corner square 1, Peter replies with the corner square 9 . . . . .	89
64. Results of John's first move 1 . . . . .	89
65. John starts with the border square 2, Peter replies with the corner square 1 . . . . .	90
66. John starts with the border square 2, Peter replies with the border square 4 . . . . .	90
67. John starts with the border square 2, Peter replies with the corner square 7 . . . . .	91
68. John starts with the border square 2, Peter replies with the border square 8 . . . . .	91
69. Results of John's first move 2 . . . . .	92
<i>VIII. Modification of the Game of Noughts and Crosses</i>	
70. First modification of the game . . . . .	92
71. Second modification of the game . . . . .	93
72. Conclusions from the trees of §71 . . . . .	95
<i>IX. Puzzles Derived from the Game</i>	
*73. Possible double threats by John . . . . .	96
*74. Possible double threats by Peter . . . . .	98
*75. Some more special puzzles . . . . .	98
*76. Possible cases of a treble threat . . . . .	99
77. Remark on the treble threat . . . . .	100
<b>Chapter IV: Number Systems</b>	
<i>I. Counting</i>	
78. Verbal counting . . . . .	101
79. Numbers in written form . . . . .	101
80. Concept of a digital system . . . . .	102
<i>II. Arithmetic</i>	
81. Computing in a digital system . . . . .	103
*82. Changing to another number system . . . . .	105
<i>III. Remarks on Number Systems</i>	
83. The only conceivable base of a number system is 10 . . . . .	107
84. Comparison of the various digital systems . . . . .	108
85. Arithmetical prodigies . . . . .	110
<i>IV. More about Digital Systems</i>	
86. Origin of our digital system . . . . .	111
87. Forerunners of a digital system . . . . .	111
88. Grouping objects according to a number system . . . . .	112
<b>Chapter V: Some Puzzles Related to Number Systems</b>	
<i>I. Weight Puzzles</i>	
89. Bachet's weights puzzle . . . . .	115
90. Weights puzzles with weights on both pans . . . . .	116
91. Relation to the ternary system . . . . .	118
<i>II. Example of a Binary Puzzle</i>	
92. Disks puzzle . . . . .	119
93. Origin of the disks puzzle . . . . .	121
<i>III. Robuse and Related Binary Puzzles</i>	
94. Robuse . . . . .	121

	page
95. Transposition puzzles . . . . .	126
*96. Other transposition puzzles . . . . .	129
 <b>Chapter VI: Games with Piles of Matches</b>	
<i>I. General Observations</i>	
97. General remarks . . . . .	131
98. Winning situations . . . . .	131
<i>II. Games with One Pile of Matches</i>	
99. Simplest match game . . . . .	132
100. Extension of the simplest match game . . . . .	133
101. More difficult game with one pile of matches . . . . .	134
<i>III. Games with Several Piles of Matches</i>	
102. Case of two piles . . . . .	137
103. Case of more than two piles and a maximum of 2 . . . . .	137
104. Case of more than two piles and a maximum of 3 . . . . .	138
*105. Case of more than two piles and a maximum of 4 or 5 . . . . .	139
*106. As before, but the last match loses . . . . .	140
<i>IV. Some Other Match Games</i>	
107. Game with two piles of matches . . . . .	140
108. Game with three piles of matches . . . . .	141
*109. Extension to four or five piles . . . . .	141
*110. Modification of the game with three piles of matches . . . . .	142
111. Match game with an arbitrary number of piles . . . . .	142
*112. Case in which loss with the last match is a simpler game . . . . .	143
<i>V. Game of Nim</i>	
113. General remarks . . . . .	144
114. Game of nim with two piles . . . . .	144
115. Some winning situations . . . . .	144
<i>VI. Game of Nim and the Binary System</i>	
116. Relation to the binary system . . . . .	145
117. Proof of the rule for the winning situations . . . . .	146
118. Remarks on the correct way of playing . . . . .	147
119. Case in which the last match loses . . . . .	148
120. Simplest way to play . . . . .	148
<i>VII. Extension or Modification of the Game of Nim</i>	
121. Extension of the game of nim to more than three piles . . . . .	150
*122. Further extension of the game of nim . . . . .	151
*123. Special case of the game of §122 . . . . .	152
*124. Modification of the game of nim . . . . .	153
 <b>Chapter VII: Enumeration of Possibilities and the Determination of Probabilities</b>	
<i>I. Numbers of Possibilities</i>	
125. Multiplication . . . . .	155
126. Number of complete permutations . . . . .	156
127. Number of restricted permutations . . . . .	157
128. Number of combinations . . . . .	158
129. Number of permutations of objects, not all different . . . . .	159
130. Number of divisions into piles . . . . .	160

	page
<i>II. Determining Probabilities from Equally Likely Cases</i>	
131. Notion of probability . . . . .	162
132. Origin of the theory of probability . . . . .	163
133. Misleading example of an incorrect judgment of equal likelihood . . . . .	166
<i>III. Rules for Calculating Probabilities</i>	
134. Probability of either this or that; the addition rule . . . . .	167
135. Probability of both this and that; the product rule . . . . .	169
136. Examples of dependent events . . . . .	170
137. Maxima and minima of sequences of numbers . . . . .	171
138. Extension to several events . . . . .	172
139. Combination of the sum rule and product rule . . . . .	173
140. More about maxima and minima in a sequence of numbers . . . . .	174
<i>IV. Probabilities of Causes</i>	
141. A posteriori probability: the quotient rule . . . . .	176
142. Application of the quotient rule . . . . .	177
143. Another application . . . . .	178
<b>Chapter VIII: Some Applications of the Theory of Probability</b>	
<i>I. Various Questions on Probabilities</i>	
144. Shrewd prisoner . . . . .	180
145. Game of kasje . . . . .	181
*146. Simplification of the game of kasje . . . . .	182
147. Poker dice . . . . .	186
148. Probabilities in poker dice . . . . .	187
<i>II. Probabilities in Bridge</i>	
149. Probability of a given distribution of the cards . . . . .	189
150. A posteriori probability of a certain distribution of the cards . . . . .	192
151. Probabilities in finessing . . . . .	194
<b>Chapter IX: Evaluation of Contingencies and Mean Values</b>	
<i>I. Mathematical Expectation and Its Applications</i>	
152. Mathematical expectation . . . . .	198
153. Examples of mathematical expectation . . . . .	198
154. More complicated example . . . . .	199
155. Modification of the example of §154 . . . . .	201
156. Petersburg paradox . . . . .	201
<i>II. Further Application of Mathematical Expectation</i>	
157. Application of mathematical expectation to the theory of probability . . . . .	203
158. Law of large numbers . . . . .	205
159. Probable error . . . . .	206
160. Remarks on the law of large numbers . . . . .	207
161. Further relevance of the law of large numbers . . . . .	208
<i>III. Average Values</i>	
162. Averages . . . . .	208
163. Other examples of averages . . . . .	210
164. Incorrect conclusion from the law of large numbers . . . . .	211

	page
<b>Chapter X: Some Games of Encirclement</b>	
<i>I. Game of Wolf and Sheep</i>	
165. Rules of the game of wolf and sheep . . . . .	214
166. Correct methods for playing wolf and sheep . . . . .	215
167. Some wolf and sheep problems . . . . .	217
168. Even and odd positions . . . . .	220
169. Final remark on wolf and sheep . . . . .	221
<i>II. Game of Dwarfs or "Catch the Giant!"</i>	
170. Rules of the game . . . . .	222
171. Comparison with wolf and sheep . . . . .	223
172. Remarks on correct lines of play . . . . .	224
173. Correct way of playing . . . . .	225
174. Winning positions . . . . .	226
175. Positions where the dwarfs are to move . . . . .	228
<i>III. Further Considerations of the Game of Dwarfs</i>	
176. Remarks on diagrams D, E, and G . . . . .	230
177. Critical positions . . . . .	230
178. More about the correct way of playing . . . . .	231
179. Trap moves by the giant . . . . .	231
180. Comparison of the game of dwarfs with chess . . . . .	233
<i>IV. Modified Game of Dwarfs</i>	
181. Rules of the game . . . . .	234
182. Winning positions of the modified game . . . . .	235
183. Case in which the dwarfs have to move . . . . .	236
184. Dwarfs puzzle . . . . .	237
185. Remark on diagrams A-H . . . . .	238
186. Other opening moves of the giant . . . . .	239
<i>V. The Soldiers' Game</i>	
187. Rules of the game . . . . .	239
188. Winning positions . . . . .	240
189. Course of the game . . . . .	241
190. Other winning positions . . . . .	242
191. Modified soldiers' game . . . . .	243
<b>Chapter XI: Sliding-Movement Puzzles</b>	
<i>I. Game of Five</i>	
192. Rules of the game . . . . .	245
193. Some general advice . . . . .	246
194. Moving a single cube . . . . .	246
195. Condition for solvability . . . . .	247
<i>II. Extensions of the Game of Five</i>	
196. Some results summarized . . . . .	247
197. Proof of the assertions of §196 . . . . .	248
<i>III. Fatal Fifteen</i>	
198. Further extension of the game of five . . . . .	249
199. Proof of corresponding results . . . . .	250
<i>IV. Further Considerations on Inversions</i>	
200. Property of inversions . . . . .	251
*201. Cyclic permutation . . . . .	252
*202. Parity determination in terms of cyclic permutations . . . . .	252

*V. Least Number of Moves*

203. Determination of the least number of moves . . . . .	253
204. First example . . . . .	254
*205. Some more examples . . . . .	256

*VI. Puzzles in Decanting Liquids*

206. Simple decanting puzzle with three jugs . . . . .	257
207. Another decanting puzzle with three jugs . . . . .	258
208. Remarks on the puzzles of §§206 and 207 . . . . .	258
209. Changes of the three jugs . . . . .	259
210. Further remarks on the three-jug puzzle . . . . .	259
211. Decanting puzzle with four jugs . . . . .	260
212. Another puzzle with four jugs . . . . .	261

**Chapter XII: Subtraction Games**

*I. Subtraction Game with a Simple Obstacle*

213. Subtraction games in general . . . . .	263
214. Subtraction games with obstacles . . . . .	264
215. Winning numbers when 0 wins . . . . .	264
216. Winning numbers when 0 loses . . . . .	265

*II. Subtraction Game with a More Complicated Obstacle*

217. Rules of the game . . . . .	266
218. Even-subtraction game . . . . .	267
219. Odd-subtraction game . . . . .	268

*III. 3-, 5-, 7- and 9-Subtraction Games*

220. 3-subtraction game . . . . .	268
221. The other 3-subtraction games . . . . .	269
222. 5-subtraction game . . . . .	270
223. 7-subtraction game . . . . .	272
224. 9-subtraction game . . . . .	274

*IV. Subtraction Game where the Opener Loses*

225. Modified subtraction game . . . . .	275
226. Modified 2-, 3-, 4-, and 5-subtraction games . . . . .	276
227. Modified 6-, 7-, 8-, and 9-subtraction games . . . . .	227
*228. Modified subtraction game with larger deductions . . . . .	278

**Chapter XIII: Puzzles with Some Mathematical Aspects**

*I. Simple Puzzles with Squares*

229. Puzzle with two square numbers of two or three digits . . . . .	281
230. Puzzle with three 3-digit squares . . . . .	282
231. Puzzle of §230 with initial zeros . . . . .	283

*II. Puzzle with 4-Digit Squares*

232. 4-digit squares . . . . .	284
233. Puzzle of the four 4-digit squares . . . . .	285
234. Puzzle of §233 with zeros . . . . .	286

*III. A Curious Multiplication*

235. Multiplication puzzle with 20 digits . . . . .	287
236. Connection with remainders for divisions by 9 . . . . .	288
237. Combination of the results of §§235 and 236 . . . . .	289

	page
<i>IV. Problem on Remainders and Quotients</i>	
238. Arithmetical puzzle . . . . .	291
239. Variants of the puzzle of §238 . . . . .	292
*240. Mathematical discussion of the puzzle . . . . .	294
<i>V. Commuter Puzzles</i>	
241. Simple commuter puzzle . . . . .	295
242. More difficult commuter puzzle . . . . .	296
243. Solution of the puzzle of §242 . . . . .	296
<i>VI. Prime Number Puzzles</i>	
244. Prime number puzzle with 16 squares . . . . .	297
245. Solution of the puzzle of §244 . . . . .	298
246. Examination of the five cases . . . . .	301
247. Puzzle of §244 with a restriction . . . . .	302
248. Prime number puzzle with 25 squares . . . . .	303
249. Puzzle with larger prime numbers . . . . .	306
<i>VII. Remarkable Divisibility</i>	
250. Divisibility of numbers in a rectangle . . . . .	307
251. Puzzle with multiples of 7 . . . . .	308
252. Multiples of 7 puzzle with the largest sum . . . . .	309
253. Proof that the solutions found do in fact yield the largest sum . . . . .	311
254. Multiples of 7 with the maximum product . . . . .	312
<i>VIII. Multiplication and Division Puzzles</i>	
255. Multiplication puzzle "Est modus in rebus" . . . . .	314
256. Multiplication and division puzzle . . . . .	317
*257. Terminating division puzzle . . . . .	318
*258. Repeating division puzzle . . . . .	320
<i>IX. Dice Puzzles</i>	
259. Symmetries of a cube . . . . .	322
*260. Group of symmetries . . . . .	323
*261. Symmetries of the regular octahedron . . . . .	324
262. Eight dice joined to make a cube . . . . .	324
*263. More difficult puzzle with eight dice . . . . .	325
*264. Which are the invisible spot numbers? . . . . .	329
<b>Chapter XIV: Puzzles of Assorted Types</b>	
<i>I. Network Puzzle</i>	
265. Networks . . . . .	331
266. Puzzle on open and closed paths . . . . .	331
267. Relation to the vertices of the network . . . . .	332
268. Splitting a network . . . . .	333
269. Case of an odd network . . . . .	334
270. Case of two odd vertices or none . . . . .	334
271. Another determination of the smallest number of paths . . . . .	335
<i>II. Broken Lines through Dots</i>	
272. Broken line through nine dots . . . . .	336
273. Broken line through twelve dots . . . . .	336
274. Broken line through sixteen dots . . . . .	337
275. Broken line with six line segments through sixteen dots . . . . .	338
<i>III. Other Puzzles of a Geometrical Nature</i>	
276. Land division puzzles . . . . .	342

	page
277. Another division puzzle . . . . .	342
278. Polyhedron puzzles . . . . .	343
<i>IV. Queens on the Chessboard</i>	
279. Eight Queens on the chessboard . . . . .	344
280. Queens on a smaller board . . . . .	347
281. Five Queens on the chessboard . . . . .	348
<i>V. Knight on the Chessboard</i>	
282. Knight's tour . . . . .	349
283. Simple examples of closed Knight's tours . . . . .	349
284. Other closed Knight's tours . . . . .	350
*285. Angles of a closed Knight's tour . . . . .	351
286. Knights commanding the entire board . . . . .	353
<i>VI. Problem on Names</i>	
*287. Who wins what? . . . . .	354
*288. Solution of the names puzzle . . . . .	356
<i>VII. Road Puzzles</i>	
289. Simple road puzzle . . . . .	358
290. More difficult road puzzle . . . . .	359
291. Still more difficult case . . . . .	361
*292. Modification of the puzzle of §289 . . . . .	362
<i>VIII. Some Puzzles on Sums</i>	
293. Puzzle with seven sums . . . . .	363
294. Modified puzzle with seven sums . . . . .	364
295. Puzzle with twelve sums . . . . .	365
*296. Another puzzle with twelve sums . . . . .	367
*297. Knight's move puzzle with eight sums . . . . .	368
<i>IX. Counting-off Puzzles</i>	
298. Simple counting-off puzzle . . . . .	372
299. More difficult counting-off puzzle . . . . .	372
300. Modification of the puzzle of §299 . . . . .	372
301. Counting-off puzzle for 1-2-3 . . . . .	373
<i>X. Solving Counting-Off Puzzles by Reversal</i>	
302. Solution of the puzzle of §298 by reversal . . . . .	375
303. Solution of the puzzle of §299 by reversal . . . . .	376
304. Solution of the puzzle of §301 by reversal . . . . .	377
<i>XI. Stockings Puzzles and Probability Considerations</i>	
305. Stockings puzzle . . . . .	379
306. Probability problem on the stockings puzzle . . . . .	379
*307. Approximations to the probabilities of §306 . . . . .	381
308. Accuracy of Stirling's formula . . . . .	382
*309. Further applications of the results . . . . .	383
<i>XII. Miscellaneous Puzzles</i>	
310. Keys puzzle . . . . .	383
311. Puzzle on numbers . . . . .	385
312. Adding puzzle . . . . .	386
313. Number written with equal digits . . . . .	388

**Chapter XV: Puzzles in Mechanics**

*I. General Remarks on Kinematics*

314. Kinematics and dynamics . . . . .	390
--	-----

	page
315. Motion of a point along a straight line . . . . .	390
316. Motion of a point along a curved line . . . . .	391
317. Relative character of motion . . . . .	392
318. Composition of motions . . . . .	392
319. Examples of the composition of motions . . . . .	393
<i>II. Some Puzzles in Kinematics</i>	
320. Which points of a traveling train move backwards? . . . . .	396
321. Which way does the bicycle go? . . . . .	398
322. Rower and bottle . . . . .	399
323. Four points moving along straight lines . . . . .	401
<i>III. Phenomena of Inertia</i>	
324. Principle of inertia . . . . .	402
325. To what systems does the principle of inertia apply? . . . . .	403
326. Some consequences of the rotation of the Earth . . . . .	404
327. Some phenomena of inertia . . . . .	407
<i>IV. Mass and Weight</i>	
328. Mass and force . . . . .	409
329. Gravitation; weight . . . . .	410
330. Acceleration of gravity; free fall . . . . .	411
<i>V. Further Considerations on Forces</i>	
331. Centrifugal force . . . . .	413
332. Influence of centrifugal force on gravitation . . . . .	415
333. Principle of action and reaction . . . . .	417
334. Motion of the center of gravity . . . . .	418
<i>VI. Some Puzzles in Dynamics</i>	
335. Enchanted cage . . . . .	420
336. Problem of the falling elevator . . . . .	420
337. Will the body topple over onto the smooth inclined plane? . . . . .	421
338. How do I get off a smooth table? . . . . .	422
<i>VII. Some Other Dynamic Problems</i>	
339. Problem of the climbing monkey . . . . .	423
340. Extension of the problem of the climbing monkey . . . . .	424
341. Related problem . . . . .	427
*342. Some calculations concerning the problems of §341 . . . . .	429