

Table of Contents

1	An idealized world made of simple elements	5
1.1	Points, straight lines, and circles in the drawing plane	6
1.2	Special points inside the triangle	11
1.3	Elementary building blocks in space	23
1.4	Euclidean space	25
1.5	Polarity, duality and inversion	31
1.6	Projective and non-Euclidean geometry	40
2	Projections and shadows: Reduction of the dimension	47
2.1	The principle of the central projection	48
2.2	Through restrictions to parallel projection and normal projection	52
2.3	Assigned normal projections	58
2.4	The difference about technical drawing	69
3	Polyhedra: Multiple faced and multi-sided	73
3.1	Congruence transformations	74
3.2	Convex polyhedra	77
3.3	Platonic solids	86
3.4	Other special classes of polyhedra	92
3.5	Planar sections of prisms and pyramids	97
3.6	"Explorations" of the truncated octahedron	103
4	Curved but simple	109
4.1	Planar and space curves	110
4.2	The sphere	125
4.3	Cylinder surfaces	133
4.4	The ellipse as a planar intersection of a cylinder of revolution	136
5	More about conic sections and developable surfaces	145
5.1	Cone surfaces	146
5.2	Conic sections	153
5.3	General developables (torses)	166
5.4	About maps and "sphere developments"	174
5.5	The reflection in a circle, a sphere, and a cylinder of revolution	183
6	Prototypes	189
6.1	Second-order surfaces	190
6.2	Three types of surface points	207
6.3	Surfaces of revolution	216
6.4	The torus as a prototype for all other surfaces of revolution	224
6.5	Pipe and canal surfaces	232
7	Further remarkable classes of surfaces	237
7.1	Ruled surfaces	238
7.2	Helical surfaces	244
7.3	Different types of spiral surfaces	256
7.4	Translation surfaces	261
7.5	Minimal surfaces	267

8 The endless variety of curved surfaces	273
8.1 Mathematical surfaces and free-form surfaces	274
8.2 Interpolating surfaces	278
8.3 Bézier- and B-spline-curves	280
8.4 Bézier- and B-spline-surfaces	283
8.5 Surface design in a different way	286
9 Photographic image and individual perception	291
9.1 The human eye and the pinhole camera	292
9.2 Different techniques of perspective	295
9.3 Other perspectives images	308
9.4 Geometry at the water surface	321
10 Kinematics: Geometry in motion	335
10.1 The pole, around which everything revolves	336
10.2 Different mechanisms	343
10.3 Ellipse motion	355
10.4 Trochoid motion	362
11 Spatial motions	367
11.1 Motions on the sphere	368
11.2 General spatial motion	373
11.3 What is the position of the sun?	377
11.4 About minute-precise sundials for the mean time	392
12 The multitude of filling patterns	403
12.1 Periodical tilings	404
12.2 Non-periodical tilings	410
12.3 Non-Euclidean tilings	414
13 The nature of geometry and the geometry of nature	417
13.1 The geometric basic forms in nature	418
13.2 Evolution and geometry	424
13.3 Planetary paths and fish swarms	431
13.4 Scaling behavior in nature	438
13.5 Musical harmony through the eyes of geometry	442
A Geometrical free-hand drawing	445
A.1 Normal view vs. oblique view	446
A.2 Do not be afraid of curved surfaces	452
A.3 Shadows	458
A.4 Perspective sketching	460
B A geometry-based photography course	473
B.1 Focal lengths and viewing angles	474
B.2 3D-images in photography?	477
B.3 When to use which focal length?	481
B.4 Primary and secondary projection	488
B.5 From below or from above?	492
Bibliography	499
Index	501