Contents

Preface xi

1 Elements of logic 1
- 1.1 True and false statements . . . 1
- 1.2 Logical connectives and truth tables . . . 2
- 1.3 Logical equivalence . . . 7
- 1.4 Quantifiers . . . 9
- 1.5 Exercises . . . 12

2 Proofs: Structures and strategies 15
- 2.1 Axioms, theorems and proofs . . . 16
- 2.2 Direct proof . . . 17
- 2.3 Contrapositive proof . . . 19
- 2.4 Proof by contradiction . . . 20
- 2.5 Proofs of equivalent statements . . . 21
- 2.6 Proof by cases . . . 23
- 2.7 Existence proofs . . . 24
- 2.8 Proof by counterexample . . . 25
- 2.9 Proof by mathematical induction . . . 26
- 2.10 Exercises . . . 30

3 Elementary theory of sets 33
- 3.1 Axioms for set theory . . . 33
- 3.2 Inclusion of sets . . . 35
- 3.3 Union and intersection of sets . . . 38
- 3.4 Complement, difference and symmetric difference of sets . . . 41
- 3.5 Ordered pairs and the Cartesian product . . . 46
- 3.6 Exercises . . . 48

4 Functions 51
- 4.1 Definition and examples of functions . . . 51
- 4.2 Direct image, inverse image . . . 56
- 4.3 Restriction and extension of a function . . . 59
- 4.4 One-to-one and onto functions . . . 60
- 4.5 Composition and inverse functions . . . 62
- 4.6 *Family of sets and the axiom of choice . . . 66
- 4.7 Exercises . . . 69

5 Relations **73**
5.1 General relations and operations with relations 73
5.2 Equivalence relations and equivalence classes 78
5.3 Order relations 81
5.4 *More on ordered sets and Zorn's lemma 82
5.5 Exercises 87

6 Axiomatic theory of positive integers **91**
6.1 Peano axioms and addition 91
6.2 The natural order relation and subtraction 94
6.3 Multiplication and divisibility 97
6.4 Natural numbers 99
6.5 Other forms of induction 100
6.6 Exercises 103

7 Elementary number theory **105**
7.1 Absolute value and divisibility of integers 105
7.2 Greatest common divisor and least common multiple 107
7.3 Integers in base 10 and divisibility tests 112
7.4 Exercises 113

8 Cardinality: Finite sets, infinite sets **117**
8.1 Equipotent sets 117
8.2 Finite and infinite sets 119
8.3 Countable and uncountable sets 120
8.4 Exercises 123

9 Counting techniques and combinatorics **125**
9.1 Counting principles 125
9.2 Pigeonhole principle and parity 127
9.3 Permutations and combinations 128
9.4 Recursive sequences and recurrence relations 132
9.5 Exercises 135

10 The construction of integers and rationals **139**
10.1 Definition of integers and operations 139
10.2 Order relation on integers 142
10.3 Definition of rationals, operations and order 145
10.4 Decimal representation of rational numbers 149
10.5 Exercises 151

11 The construction of real and complex numbers **153**
11.1 The Dedekind cuts approach 153
11.2 The Cauchy sequences approach 162
11.3 Decimal representation of real numbers 170
11.4 Algebraic and transcendental numbers 172

11.5 Complex numbers 172
11.6 The trigonometric form of a complex number 175
11.7 Exercises 178

Bibliography **181**

Answers to select exercises **183**

Index **201**