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To the student

There are college students who have done quite well in mathematics, especially in calculus, but who find that their first semester calculus course is a somewhat traumatic experience. The main reason for this is that the mathematics that they are learning is not the same as the mathematics that they learned in high school. In high school, mathematics was often taught as a collection of rules and formulas to be memorized and applied. In college, mathematics is taught as a subject that requires understanding and proof. This book is designed to help you make the transition from high school to college mathematics. It covers the material that you need to know for your first semester calculus course, but it also includes many exercises and examples that will help you understand the concepts and develop your problem-solving skills.

In order to be able to read this book, we assume that you have some knowledge of basic algebra, divisibility of integers, trigonometry, and calculus. Many of you may have seen some of these subjects in your first two years of college. In particular, we use the notation $\mathbb{Z}$ for the set of integers $0, 1, 2, 3, \dots$; $\mathbb{Q}$ for the set of rational numbers $0, 1/2, 3/4, \dots$; $\mathbb{R}$ for the set of real numbers $0, 1/2, \sqrt{2}, \pi, \dots$; and $\mathbb{C}$ for the set of complex numbers $0, 1, i, \sqrt{2}, \pi, \dots$. We also use the notation $f: \mathbb{R} \rightarrow \mathbb{R}$ for a function f from $\mathbb{R}$ to $\mathbb{R}$. If the domain of a function is just a subset of $\mathbb{R}$, we write $f: I \rightarrow \mathbb{R}$, where I is the domain.

Throughout this book, we will emphasize the necessity of understanding the concepts and theorems, rather than just memorizing them. We will also provide many examples and exercises that will help you develop your problem-solving skills. We will also provide many examples and exercises that will help you develop your problem-solving skills. We will also provide many examples and exercises that will help you develop your problem-solving skills.

It is our hope that this book will help you understand the concepts and theorems of calculus, and that it will also provide many examples and exercises that will help you develop your problem-solving skills. We will also provide many examples and exercises that will help you develop your problem-solving skills.