

TABLE OF CONTENTS

List of Figures	6
List of Tables	9
Notation	15
Chapter 1: Introduction and State of the Art	20
1.1 RVE definition and analysis of microstructure morphology	23
1.2 Stochastic optimization methods	24
1.3 Multiscale modeling	26
1.4 Microscale modeling	27
1.5 Analysis of woven fabric composites	29
1.6 Present work	31
Chapter 2: Quantification of microstructure morphology	33
2.1 Basic concepts and hypotheses	33
2.1.1 Concept of an ensemble	34
2.1.2 Ergodic hypothesis	35
2.1.3 Statistical homogeneity	35
2.1.4 Statistical isotropy	35
2.2 Microstructure description	36
2.2.1 n -point probability functions	36
2.2.2 Lineal path function	38
2.3 Numerical evaluation of microstructural statistics	39
2.3.1 n -point probability functions	39
2.3.2 Lineal path function	41
2.3.3 Microstructural statistics for fully overlapping discs	42
2.3.4 Numerical experiments	44
2.4 Analysis of unidirectional fiber composite	45
2.4.1 Testing ergodic hypothesis	45
2.4.2 Two-point probability function	47
2.4.3 Lineal path function	48
2.5 Optimization algorithm	49
2.5.1 Algorithm description	50
2.5.2 List of operators	51
Chapter 3: Microscale modeling via periodic fields	53
3.1 Construction of the periodic unit cell	53
3.1.1 Objective function and problem definition	54
3.1.2 Golden Section Search method	55
3.1.3 Test examples	56

3.1.4	Determination of periodic unit cell	60
3.2	Elastic response of composites with periodic microstructure	62
3.2.1	Problem setting	62
3.2.2	Formulation based on strain approach	65
3.2.3	Formulation based on stress approach	66
3.2.4	Implementation issues	67
3.2.5	Numerical results	68
3.3	Residual and thermal fields	71
3.3.1	Macroscopic constitutive law	71
3.3.2	Thermal loads	72
3.3.3	Numerical results	73
3.4	Linear viscoelastic response	74
3.4.1	Macroscopic constitutive law	74
3.4.2	Numerical results	76
3.5	Nonlinear viscoelastic response	79
3.5.1	Macroscopic constitutive law	79
3.5.2	Numerical results	80
Chapter 4:	Micromechanical modeling via extended Hashin-Shtrikman variational principles	83
4.1	Body with prescribed surface displacements and eigenstresses	84
4.1.1	Variational principle	84
4.1.2	Extension to random composites	85
4.1.3	Approximate solution	86
4.2	Body with prescribed surface tractions and eigenstrains	87
4.2.1	Variational principle	88
4.2.2	Extension to random composites	89
4.2.3	Approximate solution	90
4.3	Linear elasticity and thermal strains	91
4.4	Linear viscoelasticity	93
4.4.1	Macroscopic constitutive law	93
4.4.2	Numerical results	94
4.5	Nonlinear viscoelasticity	95
4.5.1	Macroscopic constitutive law	97
4.5.2	Numerical results	97
Chapter 5:	Mesoscale modeling via periodic fields	101
5.1	Construction of the periodic unit cell	102
5.1.1	Geometry of the periodic unit cell	102
5.1.2	Objective function and problem definition	104
5.1.3	Test examples	105
5.1.4	Mesostructures with typical tow misalignments	110
5.2	Numerical evaluation of the overall response	111
5.2.1	Problem setting	111
5.2.2	Discretization	113
5.2.3	Mesh generation	114

5.2.4 Numerical examples	115
5.3 Prestress analysis on mesoscale	118
5.3.1 Geometrical model	118
5.3.2 Periodic boundary conditions	120
5.3.3 Numerical results	121
Chapter 6: Conclusions and future work	124
Bibliography	127
Appendix A: Constitutive models	150
A.1 Vectorial representation of second-order tensors	150
A.1.1 Global-local transformations	151
A.2 Linear elasticity	151
A.2.1 Generalized plane strain	153
A.3 Linear viscoelasticity	154
A.3.1 Dirichlet series expansion	155
A.3.2 Generalized Maxwell model	156
A.4 Non-linear viscoelasticity	157
A.4.1 Generalized Maxwell-like model	158
A.5 Hill's failure criterion	159
Appendix B: The Fourier transform, fundamental solutions and micro-structural matrices	160
B.1 The Fourier transform	160
B.2 The discrete Fourier transform	160
B.3 The convolution and correlation theorems	161
B.4 Fundamental solutions	161
B.5 The Fourier transform of tensors ϵ_0^* and σ_0^*	162
B.6 Evaluation of matrices $[A_{rs}]$ and $[B_{rs}]$	164
Appendix C: Justification of uncoupled multi-scale modeling	166
C.1 Multi-scale convergence	166
C.2 Setting of the problem	169
C.3 Homogenization result	169
C.4 Strain and stress controlled homogenization	174