

Contents

1	Introduction	1
1.1	Finite Element Method	1
1.2	Solution of linear equations system	1
2	Systems of linear equations	4
2.1	Finite element method and the standard discrete system	4
2.2	Sparse matrix	5
2.3	Symmetric positive definite matrix	5
2.4	Direct methods	5
2.4.1	LU decomposition	6
2.4.2	Cholesky LL^T decomposition	6
2.4.3	LDL T decomposition	7
2.4.4	QR decomposition	7
2.4.5	Pivoting	8
2.4.6	Numerical stability	9
2.5	Iterative methods	9
2.5.1	Error and residual	10
2.5.2	Conjugate Gradients	11
3	Matrix storage schemes	12
3.1	Dense matrix	12
3.2	Skyline storage	13
3.3	Compressed rows	14
3.4	Combined storage schemes	15
4	Graph theory basics	16
4.1	Undirected graph	17
4.2	Adjacency matrix	17
4.3	Graph Laplacian	17
5	Sparse matrix orderings	18
5.1	Cuthill-McKee	21
5.2	Spectral envelope reduction	22
5.3	Minimum degree	23
5.3.1	Elimination graph	23

5.3.2	Quotient graph	25
5.3.3	Symbolical factorization	26
5.3.4	Supervariables	26
5.4	Nested Dissection	27
5.4.1	Geometric partitioning	28
5.4.2	Coordinate-free methods	29
6	Sparse Direct Solvers	30
6.1	Symbolical factorization	30
6.2	Equations reordering	31
6.3	Numerical factorization	31
6.4	Triangular solutions	33
6.5	Arithmetic performance	33
6.6	Skyline solver	35
6.7	Sparse direct solver	35
6.8	Frontal solver	35
6.9	Multifrontal solver	36
7	DSS implementation in C++	37
7.1	Component model	37
7.2	DSSolver usage	38
7.2.1	LDL^T decomposition	38
7.2.2	LL^T decomposition	38
7.2.3	LU decomposition	39
7.3	DSSolver activity	39
7.4	Alignment of equations	40
7.5	Schur complement method	43
7.6	Block matrix arithmetic	45
8	FEM applications	48
8.1	Block structure of the stiffness matrix	48
8.2	Plane stress linear statics	49
8.3	3D shell linear statics	51
8.4	Block size study	53
8.5	Primal domain decomposition	54
8.5.1	Regular plane stress problem	54
8.5.2	General 3D mechanical problem	56
8.5.3	Incompressible 2D fluid flow	58
9	Conclusions	61