

Contents

1. Linear space	8
1.1. The definition of linear space. Examples	8
2. Matrices, determinants, systems of linear equations	32
2.1. The multiplication of matrices	32
2.2. Determinants	43
2.3. Systems of linear equations	56
3. Linear transformations	84
3.1. Introduction. Basic examples	84
3.2. Linear transformations given by determining the images of bases	91
3.3. The matrix of linear transformation with respect to bases	94
4. The linear space V_3 of “free” vectors and applications in geometry	109
4.1. Testing one’s memory – how does one pass from “geometric vectors” to “free” vectors?	109
4.2. Scalar product of vectors	112
4.3. The vector product in V_3	115
4.4. Linear varieties in E_3 (applications of scalar product and vector product)	117
4.5. Analytic geometry in E_3	120
5. When a system of linear equations has a non-negative solution (the Farkas lemma)?	140

6. Bilinear and quadratic forms.	
Scalar product.	147
6.1. Bilinear forms	147
6.2. Linear spaces with a scalar product	157
7. Linear spaces with a binary operation.	
Cayley algebras and quaternions.	
Clifford geometric algebras	160
8. Tensor product of linear spaces and the Kronecker product of matrices	170
9. Eigenvalues and eigenvectors of linear transformations and matrices	180
10. Systems of linear differential equations	185
10.1. Systems of linear differential equations	185
10.2. Homogeneous linear systems	186
10.3. Transforming a system of n linear differential equations	187
10.4. Nonhomogeneous systems	189
10.5. Systems of linear equations with constant coefficients — the eigenvector method	194
10.6. The algorithmic method of finding a fundamental set of solutions	211
10.7. Nonhomogeneous linear systems with a quasipolynomial right-hand side	216
10.8. Solving a differential equation by transforming it into a suitable system	220
Appendix 1. Mathematical Induction	223
Appendix 2. Polynomials and rational functions	228
12.1. Preliminaries	228
12.2. Dividing a polynomial by a polynomial	229
12.3. Horner's schema	231
12.4. Roots of polynomials (multiplicity of roots)	234
12.5. The fundamental theorem of algebra and its consequences	239
12.6. Decomposition of a rational function into partial fractions	242

12.7. The proof of the decomposition theorem	250
Appendix 3. The fundamental theorem of algebra	256

This textbook contains 7 extended exercises, notes of the lecture held in the Czech Technical University of Prague. It contains basic linear algebra needed in engineering courses for a year degree program, and the content of the theory is supplemented with illustrative examples. Problems for individual study are formulated at the end of each chapter. The approach is - mathematical induction and elementary theory of polynomials and rational functions - and provided. The only prerequisite for reading this textbook is elementary algebra and geometry in the extent of the secondary school curriculum (9th).

Bibliography is included at the end of the textbook. There is also my acknowledgement of the help extended to me by my colleagues and students.

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