Contents

1. Linear space **8**

1.1. The definition of linear space. Examples 8

2. Matrices, determinants, systems of linear equations **32**

2.1. The multiplication of matrices 32

2.2. Determinants 43

2.3. Systems of linear equations 56

3. Linear transformations **84**

3.1. Introduction. Basic examples 84

3.2. Linear transformations given by determining the images of bases 91

3.3. The matrix of linear transformation with respect to bases 94

4. The linear space V_3 of "free" vectors and applications in geometry **109**

4.1. Testing one's memory – how does one pass from "geometric vectors" to "free" vectors? 109

4.2. Scalar product of vectors 112

4.3. The vector product in V_3 115

4.4. Linear varieties in E_3 (applications of scalar product and vector product) 117

4.5. Analytic geometry in E_3 120

5. When a system of linear equations has a non-negative solution (the Farkas lemma)? **140**

6. Bilinear and quadratic forms. Scalar product. 147

6.1. Bilinear forms 147

6.2. Linear spaces with a scalar product 157

7. Linear spaces with a binary operation. Cayley algebras and quaternions. Clifford geometric algebras 160

8. Tensor product of linear spaces and the Kronecker product of matrices 170

9. Eigenvalues and eigenvectors of linear transformations and matrices 180

10. Systems of linear differential equations 185

10.1. Systems of linear differential equations 185

10.2. Homogeneous linear systems 186

10.3. Transforming a system of n linear differential equations 187

10.4. Nonhomogeneous systems 189

10.5. Systems of linear equations with constant coefficients — the eigenvector method 194

10.6. The algorithmic method of finding a fundamental set of solutions 211

10.7. Nonhomogeneous linear systems with a quasipolynomial right-hand side 216

10.8. Solving a differential equation by transforming it into a suitable system 220

Appendix 1. Mathematical Induction 223

Appendix 2. Polynomials and rational functions 228

12.1. Preliminaries 228

12.2. Dividing a polynomial by a polynomial 229

12.3. Horner's schema 231

12.4. Roots of polynomials (multiplicity of roots) 234

12.5. The fundamental theorem of algebra and its consequences 239

12.6. Decomposition of a rational function into partial fractions 242

12.7. The proof of the decomposition theorem 250

Appendix 3. The fundamental theorem of algebra **256**