

# Contents

3.1	The Element of Arc-length on $\mathbb{H}$ ..... 3.2
3.3	Left Möbius Space ..... 3.4
3.4	From Arc-length to Metric ..... 3.5
3.5	Isometries for Hyperbolic Distance in $\mathbb{H}$ ..... 3.6
3.6	Isometries ..... 3.7
3.7	Möbius Isometries of $(\mathbb{H}, d_{\mathbb{H}})$ ..... 3.8
3.8	Pisot Model of the Hyperbolic Plane ..... 3.9
3.9	The Poincaré Disc Model ..... 3.10
3.10	A Crossed Configuration ..... 3.11
3.11	A Geometric Configuration ..... 3.12
3.12	The Definition of the Action ..... 3.13
3.13	Properties of the Action ..... 3.14
3.14	Conformality of Elements of Möb ..... 3.15
3.15	Preserving $\mathbb{H}$ ..... 3.16
3.16	Transitivity Properties of Möb( $\mathbb{H}$ ) ..... 3.17
3.17	The Geometry of the Action of Möb( $\mathbb{H}$ ) ..... 3.18
4.1	Preamble to the Second Edition ..... vii
4.2	Preamble to the First Edition ..... ix
1.1	1. The Basic Spaces ..... 1
1.2	1.1 A Model for the Hyperbolic Plane ..... 1
1.3	1.2 The Riemann Sphere $\overline{\mathbb{C}}$ ..... 8
1.4	1.3 The Boundary at Infinity of $\mathbb{H}$ ..... 18
2.1	2. The General Möbius Group ..... 23
2.2	2.1 The Group of Möbius Transformations ..... 23
2.3	2.2 Transitivity Properties of Möb <sup>+</sup> ..... 30
2.4	2.3 The Cross Ratio ..... 36
2.5	2.4 Classification of Möbius Transformations ..... 39
2.6	2.5 A Matrix Representation ..... 42
2.7	2.6 Reflections ..... 48
2.8	2.7 The Conformality of Elements of Möb ..... 53
2.9	2.8 Preserving $\mathbb{H}$ ..... 56
2.10	2.9 Transitivity Properties of Möb( $\mathbb{H}$ ) ..... 62
2.11	2.10 The Geometry of the Action of Möb( $\mathbb{H}$ ) ..... 65

<b>3. Length and Distance in <math>\mathbb{H}</math></b> .....	73
3.1 Paths and Elements of Arc-length .....	73
3.2 The Element of Arc-length on $\mathbb{H}$ .....	80
3.3 Path Metric Spaces .....	88
3.4 From Arc-length to Metric .....	92
3.5 Formulae for Hyperbolic Distance in $\mathbb{H}$ .....	99
3.6 Isometries .....	103
3.7 Metric Properties of $(\mathbb{H}, d_{\mathbb{H}})$ .....	108
<b>4. Planar Models of the Hyperbolic Plane</b> .....	117
4.1 The Poincaré Disc Model .....	117
4.2 A General Construction .....	130
<b>5. Convexity, Area, and Trigonometry</b> .....	145
5.1 Convexity .....	145
5.2 Hyperbolic Polygons .....	154
5.3 The Definition of Hyperbolic Area .....	164
5.4 Area and the Gauss–Bonnet Formula .....	169
5.5 Applications of the Gauss–Bonnet Formula .....	174
5.6 Trigonometry in the Hyperbolic Plane .....	181
<b>6. Nonplanar models</b> .....	189
6.1 The Hyperboloid Model of the Hyperbolic Plane .....	189
6.2 Higher Dimensional Hyperbolic Spaces .....	209
<b>Solutions to Exercises</b> .....	217
<b>References</b> .....	265
<b>List of Notation</b> .....	269
<b>Index</b> .....	273