

Contents

Preface	vii
1 The How, When, and Why of Mathematics	1
Spotlight: George Pólya	8
Tips on Doing Homework	11
2 Logically Speaking	13
3 Introducing the Contrapositive and Converse	25
4 Set Notation and Quantifiers	33
Tips on Quantification	45
5 Proof Techniques	47
Tips on Definitions	56
6 Sets	59
Spotlight: Paradoxes	67
7 Operations on Sets	73
8 More on Operations on Sets	81
9 The Power Set and the Cartesian Product	89
Tips on Writing Mathematics	98
10 Relations	101
Tips on Reading Mathematics	110
11 Partitions	111
Tips on Putting It All Together	119
12 Order in the Reals	121

13	Consequences of the Completeness of \mathbb{R}	133
	Tips: You Solved It. Now What?	140
14	Functions, Domain, and Range	143
	Spotlight: The Definition of Function	151
15	Functions, One-to-One, and Onto	157
16	Inverses	167
17	Images and Inverse Images	181
	Spotlight: Minimum or Infimum?	187
18	Mathematical Induction	193
19	Sequences	209
20	Convergence of Sequences of Real Numbers	223
21	Equivalent Sets	235
22	Finite Sets and an Infinite Set	243
23	Countable and Uncountable Sets	251
24	The Cantor–Schröder–Bernstein Theorem	261
	Spotlight: The Continuum Hypothesis	270
25	Metric Spaces	277
26	Getting to Know Open and Closed Sets	289
27	Modular Arithmetic	301
28	Fermat’s Little Theorem	315
	Spotlight: Public and Secret Research	320
29	Projects	325
	Tips on Talking about Mathematics	325
	29.1 Picture Proofs	327
	29.2 The Best Number of All (and Some Other Pretty Good Ones)	330
	29.3 Set Constructions	332
	29.4 Rational and Irrational Numbers	334
	29.5 Irrationality of e and π	336
	29.6 A Complex Project	338
	29.7 When Does $f^{-1} = 1/f$?	342
	29.8 Pascal’s Triangle	343
	29.9 The Cantor Set	346

29.10 The Cauchy–Bunyakovsky–Schwarz Inequality	349
29.11 Algebraic Numbers	351
29.12 The Axiom of Choice	353
29.13 The RSA Code	357
Spotlight: Hilbert’s Seventh Problem	359
Appendix	363
Algebraic Properties of \mathbb{R}	363
Order Properties of \mathbb{R}	364
Axioms of Set Theory	364
Pólya’s List	366
References	367
Index	371