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Design of experiments is a powerful tool for understanding systems and processes. In practice, this understanding often leads immediately to improvements. We present optimal design of experiments as a general and flexible method for applying design of experiments. Our view is that optimal design of experiments is an appropriate tool in virtually any situation that suggests the possible use of design of experiments.

Books on application areas in statistics or applied mathematics, such as design of experiments, can present daunting obstacles to the nonexpert. We wanted to write a book on the practical application of design of experiments that would appeal to new practitioners and experts alike. This is clearly an ambitious goal and we have addressed it by writing a different kind of book.

Each chapter of the book contains a case study. The presentation of the case study is in the form of a play where two consultants, Brad and Peter, of the (fictitious) Intrepid Stats consulting firm, help clients in various industries solve practical problems. We chose this style to make the presentation of the core concepts of each chapter both informal and accessible.

This style is by no means unique. The use of dialogs dates all the way back to the Greek philosopher Plato. More recently, Galileo made use of this style to introduce scientific ideas. His three characters were: the teacher, the experienced student, and the novice.

Though our case studies involve scripted consulting sessions, we advise readers not to copy our consulting style when collaborating on their own design problems. In the interest of a compact exposition of the key points of each case, we skip much of the necessary information gathering involved in competent statistical consulting and problem solving.

We chose our case studies to show just how general and flexible the optimal design of experiments approach is. We start off by a chapter dealing with a simple comparative experiment. The next two chapters deal with a screening experiment and a follow-up experiment in a biotechnology firm. In Chapter 4, we show how a designed response surface experiment contributes to the development of a robust production process in food packaging. In Chapter 5, we set up a response surface experiment to maximize the yield of a chemical extraction process. Chapter 6 deals with an experiment, similar in structure to mixture experiments in the chemical and pharmaceutical industries, aimed at improving the finishing of aluminum sheets. In Chapters 7 and 8, we apply the optimal design of experiments approach to a vitamin