

This largely self-contained book on the theory of quantum information focuses on precise mathematical formulations and proofs of fundamental facts that form the foundation of the subject. It is intended for graduate students and researchers in mathematics, computer science, and theoretical physics seeking to develop a thorough understanding of key results, proof techniques, and methodologies that are relevant to a wide range of research topics within the theory of quantum information and computation.

The book is accessible to readers with an understanding of basic mathematics, including linear algebra, mathematical analysis, and probability theory. An introductory chapter summarizes these necessary mathematical prerequisites, and starting from this foundation the book includes clear and complete proofs of all results it presents. Each subsequent chapter includes challenging exercises intended to help readers develop their own skills for discovering proofs concerning the theory of quantum information.

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“This book serves not only as a beautiful introduction to quantum information, but also to the most important techniques for reasoning about it. By introducing representation theory, semidefinite programming, measure concentration, and other important tools on the way to presenting many quantum information highlights, Watrous equips the reader with the skills necessary to contribute at the research frontier.”

Patrick Hayden, *Stanford University, California*

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