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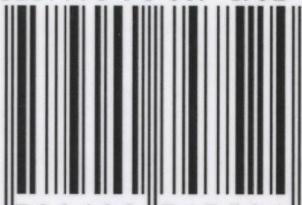
Unbounded Self-adjoint Operators on Hilbert Space

The book is a graduate text on unbounded self-adjoint operators on Hilbert space and their spectral theory with the emphasis on applications in mathematical physics (especially, Schrödinger operators) and analysis (Dirichlet and Neumann Laplacians, Sturm-Liouville operators, Hamburger moment problem). Among others, a number of advanced special topics are treated on a text book level accompanied by numerous illustrating examples and exercises. The main themes of the book are the following:

- Spectral integrals and spectral decompositions of self-adjoint and normal operators
- Perturbations of self-adjointness and of spectra of self-adjoint operators
- Forms and operators
- Self-adjoint extension theory: boundary triplets, Krein-Birman-Vishik theory of positive self-adjoint extensions.

Mathematics

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