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This began to change in the 1970s and 1980s, when a few pioneers were inspired to ask whether some of the fundamental questions of computer science and information theory could be applied to the study of quantum systems. Instead of looking at quantum systems purely as phenomena to be explained as they are found in nature, they looked at them as systems that can be designed. This seems a small change in perspective, but the implications are profound. No longer is the quantum world taken merely as presented, but instead it can be created. The result was a new perspective that inspired both a resurgence of interest in the fundamentals of quantum mechanics, and also many new questions combining physics, computer science, and information theory. These include questions such as: what are the fundamental physical limitations on the space and time required to construct a quantum state? How much time and space are required for a given dynamical operation? What makes quantum systems difficult to understand and simulate by conventional classical means?

Writing this book in the late 1990s, we were fortunate to be writing at a time when these and other fundamental questions had just crystallized out. Ten years later it is clear such questions offer a sustained force encouraging a broad research program at the foundations of physics and computer science. Quantum information science is here to stay. Although the theoretical foundations of the field remain similar to what we discussed 10 years ago, detailed knowledge in many areas has greatly progressed. Originally, this book served as a comprehensive overview of the field, bringing readers near to the forefront of research. Today, the book provides a basic foundation for understanding the field, appropriate either for someone who desires a broad perspective on quantum information science, or an entryway for further investigation of the latest research literature. Of course,